Adaptive multipath mitigation algorithm for GPS C/A code tracking loop

QIN Xinxian[1,2,3], **HAN Chengde**[1], **XIE Yingke**[1]

1. *Institute of Computing Technology*, *Chinese Academy of Science*, *Beijing* 100190, *China*;
2. *College of Computer Science and Electronic Information*, *Guangxi University*, *Guangxi Nanning* 530001, *China*;
3. *Graduate University of the Chinese Academy of Sciences*, *Bejing* 100190, *China*

Abstract: Multipath not only degrades the sensitivity of a GPS receiver, but also becomes the dominant error resource in GPS positioning systems. Multipath mitigation algorithm is the most significant in high sensitive and high accuracy GPS receivers. In this paper, multipath signal model in code tracking loop is studied first and a novel method for mitigation of multipath is presented which employs adaptive filtering recursive least square (RLS) algorithm. The system parameters need not be estimated in adaptive filtering algorithm and multipath signal can be filtered out directly. Under noisy scenario, the RLS algorithm is the best estimate for the filter weights with minimum least-squares errors. Simulation results show that the proposed method can reduce multipath error envelope effectively. The crossing-zero bias of the discriminator can be corrected which enhances the accuracy of code tracking in GPS receiver's delay-locked loop (DLL). The most importance is that the RLS algorithm is recursive and convenient to implement both in hardware and software.

Key words: GPS, multipath, tracking loop, adaptive filter, error envelope

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1 INTRODUCTION

Due to its high positioning accuracy and round-the-clock services, GPS satellite system has been extensively used. The factors that affect the positioning accuracy include: ionosphere delay, troposphere delay, ephemeris error, and receiver noise and multipath (James, 2004). Among these errors, the multipath is the one changing with the environment parameters and other errors can be revised by theoretical model or empirical formulas to the extent of toleration. If the DGPS technique is used, these errors can be further reduced. So the multipath is becoming the hottest issue in GPS receiver research.

There are several categories of multipath mitigation algorithm: spatial signal processing technique based on antenna, receiver baseband signal processing and post data processing.

Spatial signal processing technique based on antenna changes the antenna pattern to cope with the multipath issue. These techniques includes: special antenna, multiple antenna array spatial signal processing, special antenna posing strategy and obtaining the environment reflection parameters by long term observation to adjust the geometry characteristics. Unfortunately all the methods based on antenna have disadvantages of bulky and heavy volume. The worst is that none of them can perfectly mitigate the multipath higher than horizontal level (Weill, 1997).

Receiver baseband signal processing is the most flexible and effective method to mitigate multipath. Narrow correlation with the spacing between early and late correlators less than 0.1 chip was proposed to effectively mitigate multipath effects (Fenton, 1991; Van Dierendonck, 1992). The narrow correlation method employs bigger IF bandwidth which is effective to mitigate the long delay multipath. Multipath estimation technique (MET) using the slope of the autocorrelation function to estimate the code phase offset delay of the direct signal can greatly improve the mitigation performance (Townsend & Fenton, 1994). Multipath estimation delay lock loop (MEDLL) using multiple correlators to separate the incoming signal into its line-of-sight (LOS) and multipath components can achieve much more pure LOS (VanNee, 1995). Moelker (1997) first proposed Multiple Signal Classification (MUSIC) to mitigate multipath by using multiple antennas and extended MEDLL techniques. Edge correlator (Garin, 1996) is more effective to mitigate long delay multipath comparing to narrow correlators. Strobe correlator and the enhanced strobe correlator techniques (Garin & Rousseau, 1997) employing multiple correlators to make up a strobe

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First author biography: QIN Xinxian (1963—), male. He received his B.A's and M.A's degree in radio engineering from Tsinghua University in 1985 and 1990 respectively, and has become a Ph.D candidate in computing science in Graduate University of the Chinese Academy of Sciences. His research interests include weak signal processing, wireless communication and embedded system. E-mail: qinxinxian@ict.ac.cn

correlator have better performance to cope with long delay multipath. Stansell and Maenpa (1999) proposed a method called ClearTrack™ which can effectively mitigate code multipath. Its maximum code phase error is less than one quarter of narrow correlator. Nevertheless, in some environments such as that there is strong multipath or environment changes fast, none of these method can work effectively.

Post data processing techniques is also an efficient method to mitigate multipath. This method employs the received data to establish error model, so it is difficult to be used in real time positioning system.

Obviously, receiver baseband signal processing is the most flexible and effective method to mitigate multipath. In order to deal with short delay multipath (such as indoor environment) and fast changing scenario, we proposed an algorithm by utilizing adaptive filter to mitigate multipath. This algorithm is effective to mitigate short delay multipath.

2 MODEL OF MULTIPATH SIGNAL

The effects of multipath are aroused by scatter and reflection of the terrain and physiognomy when the satellite signal is transmitted to receiver (Soubielle *et al.*, 2002).

Scatter signal is negligible in GPS receiver because of its low energy. So the multipath signal is primarily composed of reflection signal. Based on this assumption, GSP satellite signal can be regarded as a summation signal of different delay time and different attenuate reflection signals (Mohammad, 2005; Gadallah, 1998).

2.1 LOS (Line Of Sight)

GPS satellites utilize CDMA technique to transmit signals which consist of PRN, navigation data and carrier to all users simultaneously in circumstance with various interferences. PRN code includes Precision Code (P Code) and Coarse Code (C/A Code). P Code serves for authorised users and C/A code serves for civilian users. Hereinafter, C/A is discussed except special explanation.

C/A Code signal can be expressed as:

$$S(t) = A\alpha_0 D(t) C(t) \cos(\omega_c t + \theta_0)$$

where, α_0: attenuation coefficient, for LOS, α_0=1

A: Amplitude of LOS

θ_0: Carrier phase of LOS

ω_c: Carrier frequency

$C(t)$: C/A Code

$D(t)$: Navigation data

It is assumed that navigation data does not change in tracking loop, so $D(t)$=1 and the expression can be simplified as:

$$S(t) = A\alpha_0 C(t) \cos(\omega_c t + \theta_0)$$

Complex signal expression:

$$S(t) = A\alpha_0 C(t) \mathrm{e}^{\mathrm{j}(\omega_c t + \theta_0)} \tag{1}$$

2.2 Multipath signal

If there are N paths in received signal, the expression of the resultant:

$$S_{\mathrm{mp}}(t) = \sum_{i=0}^{N} A\alpha_i C(t-\tau_i) \mathrm{e}^{\mathrm{j}(\omega_c t + \theta_i)} + n(t) \tag{2}$$

where, N: Number of multipath

α_i: i-th path reflection coefficient and α_0=1

A: Amplitude of LOS

θ_i: Carrier phase of i-th reflection signal

ω_c: Carrier frequency

$C(t-\tau_i)$: i-th delayed C/A Code and τ_0=0

$n(t)$: Addition White Gaussian Noise with zero mean (AWGN)

It is obvious that multipath signal is determined by the amplitude, phase and delayed time of reflection signal.

2.3 Local Carrier and Local C/A Code

Local carrier is generated by carrier phase lock loop (PLL). The frequency of output is the same as the received signal when PLL is locked. The complex signal expression is:

Local carrier: $F(t) = \mathrm{e}^{-\mathrm{j}(\omega_c t + \theta_c)}$

ω_c: Frequency of local carrier, it is identical with received signal when synchronization is achieved.

θ_c: Phase of local carrier

Local C/A code:

Early: $C_E(t)=C(t-\tau+\tau_d)$

Late: $C_L(t)=C(t-\tau-\tau_d)$

Prompt: $C_P(t)=C(t-\tau)$

where,: τ is the estimated value of transmitting time of C/A code, τ_d is the space between early correlator and late correlator.

2.4 Output signal of correlator

GPS system utilizes CDMA modulation technique. Navigation data is modulated by C/A Code first then the carrier is modulated by the resultant. It is reasonable that two tracking loop are implemented in GPS receiver to track carrier and C/A code signal respectively. These two loops have to cooperate with each other to track signal exactly. Because Costas loop is not sensitive to modulation signal on carrier, it is very suitable for GPS receiver carrier PLL and widely used in GPS receiver to track carrier. Another tracking loop is the C/A code tracking loop which employs Delay Lock Loop (DLL) to achieve Maximum likelihoods estimation of C/A code transmitting time. There are six correlators in code tracking loop: three in quadrature-phase arm: early correlator, prompt correlator and late correlator. Similarly three in in-phase arm: early correlator, prompt correlator and late correlator.

The received signal is separated into two arm multiplied by locally generated in-phase and quadrature-phase replicas of the carrier respectively. Then the resultants are correlated with early, prompt and late versions of the locally generated C/A code, and the correlation values are integrated for a pre-detection integration period. The early and late correlation values in the in-phase and quadrature-phase arms (I_E, I_L, Q_E, Q_L) are generally used for code tracking, whereas the prompt correlation values (I_P, Q_P) are used for carrier tracking as Fig. 1.

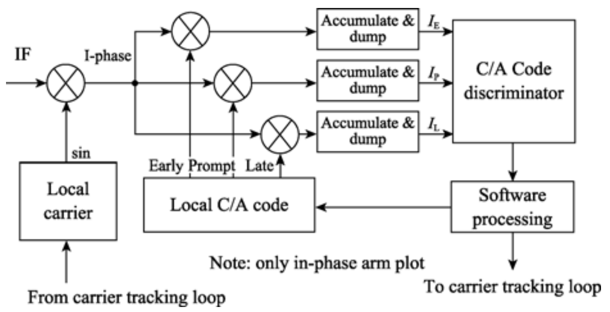

Fig. 1 In-phase arm of C/A code tracking loop DLL

In fact, the input IF signal multiplied by locally generated replicas of the carrier then multiplied by early, prompt and late versions of the locally generated C/A code, and the resultants are integrated for a pre-detection integration period. All these operations are to implement correlation. Finally the output signal is (early component):

$$I_E = E\left\{\frac{1}{T}\int_0^T S_{mp}(t)C_E(t)e^{-j(\omega_c t+\theta_c)}dt\right\}$$
$$= E\left\{\frac{1}{T}\int_0^T S_{mp}(t)C(t-\tau+\tau_d)e^{-j(\omega_c t+\theta_c)}dt\right\}$$
$$= E\left\{\frac{1}{T}\int_0^T \sum_{i=0}^{N} A\alpha_i C(t-\tau_i)e^{j(\omega_c t+\theta_i)}C(t-\tau+\tau_d)e^{-j(\omega_c t+\theta_c)}dt\right\}$$

result in:

$$I_E = \sum_{i=0}^{N} A\alpha_i \overline{C(t-\tau_i)C(t-\tau+\tau_d)}e^{j(\theta_i-\theta_c)}$$

Expressed by C/A code correlation function:

$$I_E = \sum_{i=0}^{N} H_i R(\tau-\tau_d-\tau_i) \tag{3}$$

where,

$H_i = A\alpha_i e^{j(\theta_i-\theta_c)}$, $R(\tau)=\overline{C(t)C(t-\tau)}$ is self correlation function of C/A code.

Similarly the late component:

$$I_L = \sum_{i=0}^{N} H_i R(\tau+\tau_d-\tau_i) \tag{4}$$

The output of the coherent discriminator function:

$$D_I(\tau) = I_E - I_L = \sum_{i=0}^{N} H_i[R(\tau-\tau_d-\tau_i)-R(\tau+\tau_d-\tau_i)] = \sum_{i=0}^{N} H_i g(\tau-\tau_i) \tag{5}$$

where, $g(\tau)=R(\tau-\tau_d)-R(\tau+\tau_d)$

$R(\tau)$: the self correlation function of C/A code. In the absence of multipath, the self correlation function of C/A code is a standard triangle. But in the presence of multipath the triangle becomes polygon such as Fig. 3 and Fig. 4.

$g(\tau)=R(\tau-\tau_d)-R(\tau+\tau_d)$ is the ideal discriminator function in the absence of multipath which is commonly called as S-curve. This is illustrated in Fig. 2.

3 IMPULSE RESPONSE OF MULTIPATH CHANNEL

According to Eq. (5),

$$D_I(\tau) = I_E - I_L = \sum_{i=0}^{N} H_i g(\tau-\tau_i)$$

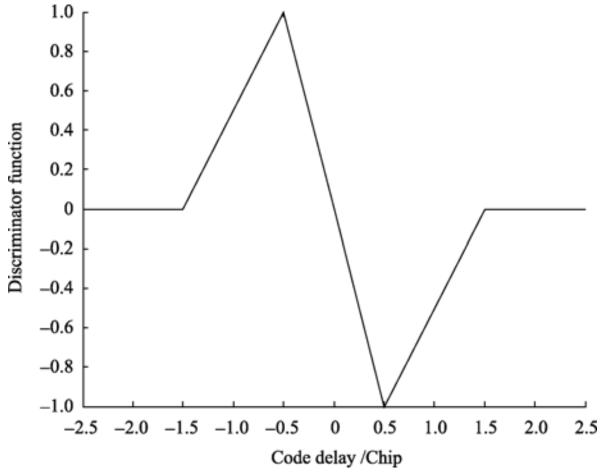

Fig. 2 Normalized S-curve of discriminator (under ideal scenario) (S-Curve: E–L=1 T_c BW=10.23MHz)

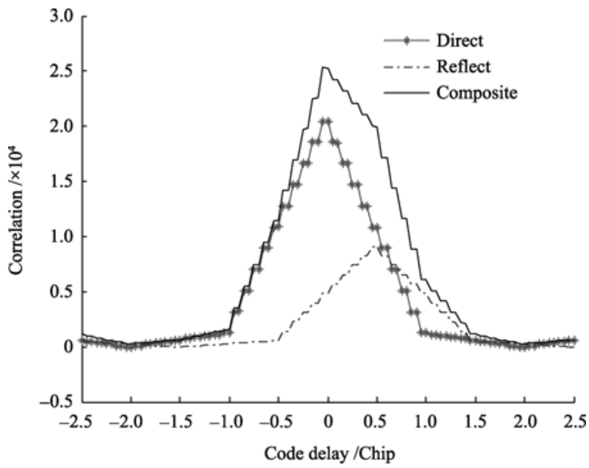

Fig. 3 One reflected signal in-phase with direct signal

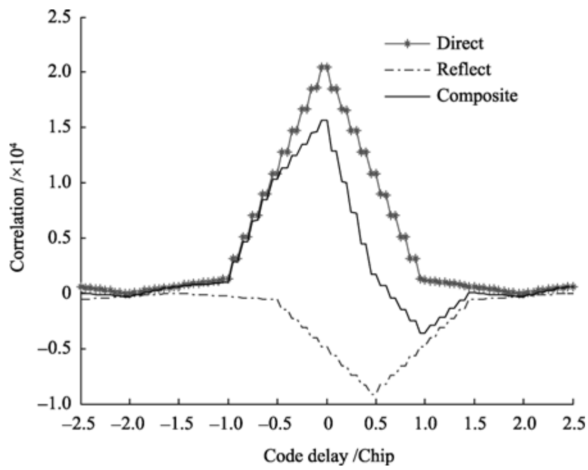

Fig. 4 One reflected signal out-phase with direct signal

Select small enough Δ enable τ and τ_d to meet the demand of precision. $\tau=m\Delta$ is the transmitting delay time to be estimated, $\tau_i=i\Delta$ is the i-ray transmitting delay time of reflected signal, the composite final output signal of discriminator can be expressed as,

$$y(m)=\sum_i h(i)g(m-i)=h(m)*g(m) \tag{6}$$

m: is the delay time of discriminator function in digital field.

i: is the number of reflected signal

* denote convolution operation

$$y(m)=D_I(m\Delta),$$

$$h(m)=H_m,$$

$$g(m-i)=g[(m-i)\Delta]$$

According to Eq. (6), after digitization the output of DLL discriminator is the convolution of $h(m)$ and $g(m)$. $h(m)$ can be regarded as the impulse response of fade channel from satellite to receiver and $g(m)$ can also be regarded as the input signal of the fade channel. The output of the fade channel is $y(m)$. The impulse response of the fade channel $h(m)$ represents the multipath effects of the environment. The model of the multipath channel can be illustrated in Fig. 5.

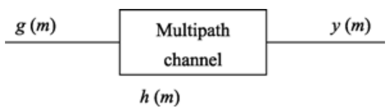

Fig. 5 Model of multipath channel

Summing-up: after the estimation of the impulse response of the multipath channel which reflects the characteristics of the multipath channel, it is possible to mitigate multipath effects using some techniques such as equalizer, inverse filter etc.

4 ADAPTIVE MULTIPATH MITIGATION

As what we have discussed, the key to mitigate the effects of multipath is to estimate the impulse response of the multipath channel $h(n)$ in accordance with certain criteria. A lot of algorithms can be used to achieve the estimation of system such as blind system estimation (Tugnait & Luo, 2002), deconvolution (Mohammad, 2005) etc. Unfortunately, all these algorithms have to do convolution operation to eliminate multipath after the estimation of system impulse $h(n)$ had been achieved such as Rlinami (2000) and Mohammad (2005).

In this paper adaptive filter is utilized to do inverse filter directly instead of estimating system impulse $h(n)$ in advance. Using adaptive filter, multipath can be easily mitigated and the shape of discrimination function of tracking loop can be rebuilt which further improves the tracking precision. As in Fig. 6, although $h(n)$ is not estimated directly, it has been worked out impliedly in inverse filtering.

4.1 Algorithm principle

The selection of reference signal is a very important issue in

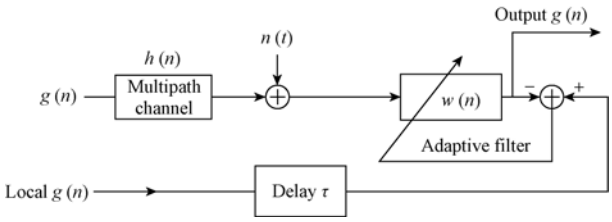

Fig. 6 Inverse filter using adaptive filter

adaptive filter. As to GPS receiver, because the discrimination function $g(n)$ of DLL is known (S-curve), it is possible to use S-curve generated by local C/A code as reference signal. The input of the adaptive filter is the received signal plus noise. The RLS algorithm can be utilized to implement adaptive filter. In practice, the RLS algorithm employs least-square theory which is the optimal estimation in finite sample space (Haykin, 2002).

Suppose that the transmitting delay of GPS signal is τ_s, the impulse response of the channel is $h(n)$, the discrimination function of receiver DLL is $g(n)$, then the input signal of the adaptive filter is,

$$x(n)=g(n-\tau_s)*h(n)+n(t)$$

The reference signal of the adaptive filter is the local discrimination function,

$$d(n)=g(n-\hat{\tau}_s)$$

$\hat{\tau}_s$: is the local estimation of transmitting delay

The estimation of discrimination function after adaptive filter can be worked out as,

$$\begin{aligned}\hat{g}(n)&=x(n)*w(n)\\&=[g(n-\tau_s)*h(n)+n(t)]*w(n)\end{aligned}$$

The error of adaptive filter,

$$\begin{aligned}e(n)&=d(n)-\hat{g}(n)\\&=g(n-\hat{\tau}_s)-\hat{g}(n)\\&=g(n-\hat{\tau}_s)-x(n)*w(n)\end{aligned} \tag{7}$$

Sum of error squares,

$$\varepsilon(n)=\sum_{i=0}^{n}e^2(i)\to\min \tag{8}$$

In Eq. (7) and Eq. (8), $x(n)=g(n-\tau_s)*h(n)+n(t)$ is the received signal which is known, and $g(n-\hat{\tau}_s)$ is the local discrimination function which is also known, then it is possible to figure out the optimal weight value $w(n)$ according to the rule of minimizing the error squares $\varepsilon(n)$. The optimal weight value $w(n)$ of the adaptive filter is the least-square solution of Eq.(8) when the filter converges. The temporal output of the adaptive filter,

$$\hat{g}(n)\approx g(n-\hat{\tau}_s)$$

$$\begin{aligned}\hat{g}(n)&=[g(n-\tau_s)*h(n)+n(t)]*w(n)\\&\approx g(n-\tau_s)*h(n)*w(n)\\&\approx g(n-\hat{\tau}_s)\end{aligned}$$

Thus $h(n)*w(n)\approx\delta(n)$

Z transform, $H(z)W(z)\approx1$

or, $W(z)\approx\dfrac{1}{H(z)}$

Now $w(n)$ can be regarded as the optimal inverse filter of channel $h(n)$ in the meaning of least-square. The output of the system input $g(n)$ passing through the concatenation of multipath channel $H(Z)$ and inverse filter $W(Z)$ is,

$$\hat{g}(n) \approx g(n-\tau_s) * h(n) * w(n) \approx g(n-\tau_s)$$

This means that $\hat{g}(n)$ is the optimal estimation of $g(n-\tau_s)$ which has mitigated the effects of multipath in the meaning of least-square.

4.2 Estimation of transmitting delay

The purpose of the code tracking loop DLL is to achieve precise estimation of transmitting delay for GPS PRN signal which can be used to get the measurement of pseudo-range.

In Eq. (7) and Eq. (8), it is possible to figure out the optimal weights of adaptive filter $w(n)$ by the given $\hat{\tau}_s$, so $w(n)$ can be regarded as a function of $\hat{\tau}_s$, and sum of the error squares $\varepsilon(n)$ is also a function of $\hat{\tau}_s$, in the range scope of $\hat{\tau}_s$:

When $\underset{\hat{\tau}_s}{\|\varepsilon(n)\|} = \min$

Corresponding $\hat{\tau}_s$ is the optimal transmitting delay.

5 SIMULATION RESULTS

5.1 S-curve revision

The lock point (equilibrium point) of the tracking loop DLL is on the zero crossing point of the discrimination function (S-curve) where τ =0. In the presence of multipath, S-curve equals the sum of direct signals and multipath signals as Eq. (6). The S-curve distorts as blue dot dash-line in Fig. 7 and offset the zeros crossing as illustrated in Fig. 7.

Here, blue dot dash-line: α_1=–3dB (0.5), code phase delay τ=1.1 Chip, carrier phase θ_1=0.15π, the space between early and late correlator τ_d=1.0 Chip, is the S-curve simulation result.

It is obvious from the simulation result of Fig. 7 that it is possible to revise the distorted S-curve to the ideal position using adaptive algorithm and zeros crossing offset decreases significantly which improves the tracking precision of the DLL remarkably.

5.2 Multipath error envelopes

In the presence of multipath, the zeros crossing point of compound signal differs from that of the direct signal. Because DLL locks on the zeros crossing point of compound signal, the difference value is the multipath error. Plot the multipath error with different delay time we get the multipath error envelope as illustrated in Fig. 8.

When multipath signal is in phase with direct signal, the error is positive forming the upper part of the envelope. When multipath signal is out of phase with the direct signal, the error is negative forming the lower part of the envelope. Fig. 8 is the simulation result of adaptive multipath mitigation algorithm.

It is obvious from Fig. 8 that after multipath mitigation the amplitude of the error envelope has been reduced greatly comparing with the foregone one. The result indicates that multipath effects have been eliminated significantly. Another merit of the algorithm is that for short delay multipath such as τ<1.8 Chip, the processing result is perfect.

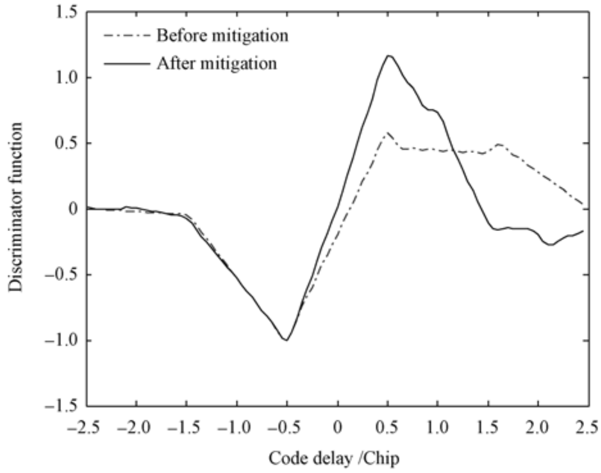

Fig. 7 S-curve in presence of multipath

(S-Curve: E–L=1 T_c, BW=10.23MHz)

5.3 Impact of signal noise rate SNR and system band width BW

Simulation result in Fig. 8, SNR=10dB, system band width BW=10MHz.

Simulation result in Fig. 9, SNR=10dB, system band width BW=2MHz. Comparing Fig. 8 with Fig. 9, it can be found that the algorithm is insensitive to system band width and it still has very excellent mitigation performance under minimum system band width, BW=2.046MHz.

Simulation result in Fig. 10, SNR= –10dB, bandwidth BW=2MHz.

The simulation results show that when the SNR is very low (SNR< –15dB), it is difficult for the adaptive filter to converge, but after SNR> –10dB, the adaptive filter converges quickly, at the same time SNR and system band width have little impact on processing results as illustrated in Fig. 8—Fig.10.

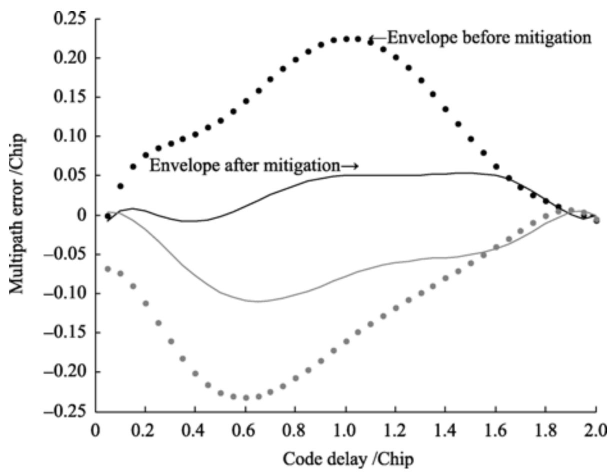

Fig. 8 Error envelope under SNR=10dB, BW=10MHz

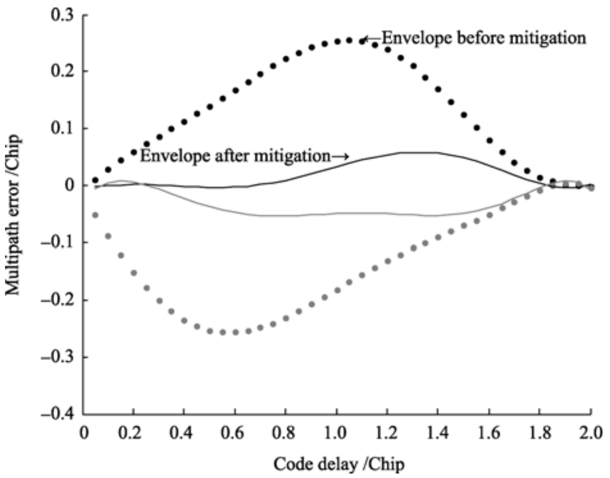

Fig. 9 Error envelope under SNR=10dB, BW=2MHz

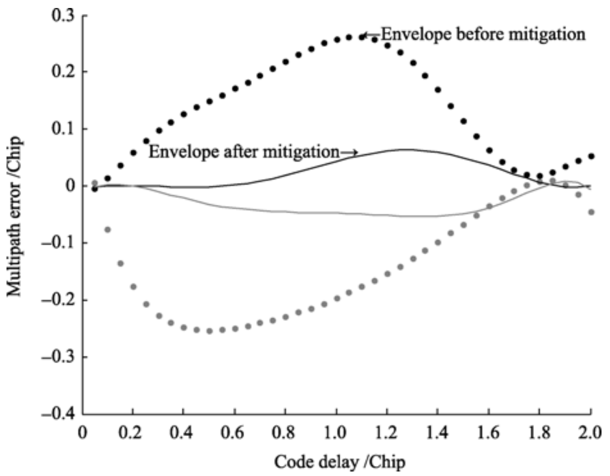

Fig. 10 Error envelope under SNR= −10dB, BW=2MHz

The reason is that if analytical method (such as least-square method) is used to achieve inverse filter, matrix inversion has to be solved and lower SNR will result in singular matrix and divergence of the adaptive filter.

In practice, our algorithm is applied after signal acquisition and the SNR of GPS signal has been improved remarkably, so it is easy to meet the requirement for SNR> −10dB.

6 DISCUSSION AND CONCLUSION

Based on the detailed study of GPS receiver code tracking loop and discriminator function S-curve, a novel algorithm for mitigation of multipath is presented which employs adaptive filtering RLS algorithm. The system parameters need not be estimated in adaptive filtering algorithm. Multipath signal can be filter out directly and original discriminator function can be rebuilt which improves the tracking precision and eliminates the effects of multipath.

Under some scenarios such as indoor positioning application where there are a lot of short delay multipath, our algorithm has some advantage over traditional one.

Simulations show that adaptive filter algorithm can eliminate the offset of zero crossing point which improves the DLL tracking precision especially for the short delay multipath. The algorithm is insensitive to system band width and SNR. Under the IF band width equals minimum BW= 2.046MHz and SNR=−10dB, the mitigation result is still perfect. Most importance is that the adaptive RLS algorithm is recursive and convenient to implement both in hardware and software.

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GPS 码跟踪环自适应多径消除算法

覃新贤[1, 2, 3]，韩承德[1]，谢应科[1]

1. 中国科学院计算技术研究所，北京 100190;
2. 广西大学计算机与电子信息工程学院，广西 南宁 530001;
3. 中国科学院 研究生院，北京 100190

摘 要： 根据多径信号的产生机理，在对GPS接收机中的码跟踪环多径信号模型研究的基础上，提出了采用自适应滤波的来消除 GPS 多径效应的算法。自适应滤波的方法不需要估计模型的系统参数，而直接通过自适应滤波将多径信号滤除。在有噪声的情况下，自适应滤波的 RLS 算法是最小二乘意义下的最优估计，仿真的结果表明采用自适应滤波算法可以快速的消除多径的影响，修正鉴相函数的过零点偏差，提高码跟踪环的跟踪精度。由于自适应滤波算法是递推算法，易于软、硬件实现。

关键词： GPS 卫星定位，多径，跟踪环，自适应滤波，误差包络

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1 引 言

GPS 卫星定位系统具有定位精度高、全天候等特点。影响 GPS 定位精度的主要因素包括: 卫星的星历误差、多普勒频移、电离层延时、对流层延时、卫星的时钟误差、接收机的时钟误差以及多径效应等(James, 2004)。对于电离层和对流层延时的估计以及卫星的钟差，采用已有的模型和经验公式，就能将其影响减小到可以容忍的程度。如果采用差分 GPS(DGPS)，误差还可以进一步减小。然而，多径效应却是随着接收环境的不同而大相径庭，因此多径问题一直是 GPS 研究中的热点问题。

消除多路径的方法大致可以分为以下几类:基于天线的空间处理方法、接收机信号处理技术和数据后处理技术。

基于天线的空间处理技术，通过改变天线的增益或天线的方向性来应对多径问题。这种方法包括: 采用特殊的天线、采用多天线阵的空间信号处理方法、特殊的天线放置策略以及通过长期的观测获取多径参数从而调整接收机周围环境的几何反射参数来消除多径。然而，基于天线的方法具有体积大、笨重的缺点，尤其致命的是，这些方法对高于水平面的多径信号，其处理效果不尽人意(Weill, 1997)。

接收机信号处理技术是消除多径最灵活、最有效的方法。窄相关器技术(Fenton, 1991; Van Dierendonck, 1992)将早和迟相关器的延时间隔减小到 0.1Chip，同时采用比较大的中频带宽，对延时大的多径消除效果比较显著。多径消除技术(Multipath Elimination Technique, MET™)实际上是窄相关器技术的改进(Townsend & Fenton, 1994)，通过估计双边自相关峰的斜率和幅度来获得峰值点的估计值。多径估计延时锁相环(Multipath Estimation Delay Lock Loop MEDLL™)利用多个窄间隔的相关器估计多径并将它从相关函数中剔除以提供更纯净的信号相关函数(VanNee, 1995)。Moelker (1997)首先描述了采用多天线和扩展MEDLL™的多信号分类技术(Multiple Signal Classification MUSIC)消除多径的方法。边沿相关器技术 (Garin, 1996)在较长延时的多径信号下具有比窄相关器技术更为优越的性能。选通相关器和增强选通相关器技术(Garin & Rousseau, 1997)采

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第一作者简介：覃新贤(1963—)，博士研究生。主要研究方向为微弱信号处理、GPS基带信号处理算法研究、无线通信系统和嵌入式技术。E-mail: qinxinxian@ict.ac.cn。

用了多个窄相关器，组合构成Strobe相关器，在对付长延时的多径信号时，也具有很好的性能。Stansell和 Maenpa (1999)提出了一种采用基于相关器的多径消除算法，称为 ClearTrack™ 算法。该算法对码多径的消除效果较好，其最大码多径误差仅为窄相关器的 1/4。但是，在多径很强而且快速变化的多径环境，这些方法消除多径的效果难以令人满意。

数据后处理技术也是消除多路径效应的一个有效办法。由于采用的是接收以后的数据，并以此建立多径误差的模型，在实时定位中很难采用。

可以看出，只有采用信号处理来消除多径是最为灵活、适用性最强的方法。在实际的应用中，为了解决短延时多径(类似在室内环境)和快速变化的多径问题，提出了采用自适应滤波器的方法消除多径的方法。这个方法对短延时多径非常有效。

2 多径信号的模型

多径效应是由于 GPS 卫星信号传输到达地面时，受到地面地形、地貌以及周围建筑物的影响产生散射和反射引起的(Soubielle 等, 2002)。

散射信号，其能量非常小，在 GPS 中，可以忽略，因此，GPS 中的多径，主要是由反射信号产生的。基于这样的假设，可以将 GPS 信号看成是多个具有不同延迟时间、不同衰减系数的反射信号的叠加(Mohammad, 2005;Gadallah, 1998)。

2.1 直达信号, LOS (Line Of Sight)

GPS 卫星采用扩频传输技术在具有各种干扰的环境下将扩频码(PRN 码)、导航数据、载波合成的信号同时从多颗卫星上传输给所有用户，其中 PRN 包括 P 码(精细码)和 C/A(粗码), C/A 码为民用码, P 码是为具有授权的特许用户服务的，以下只讨论 C/A 码。

每一颗卫星，其 C/A 码信号可以表示成:

$$S(t) = A\alpha_0 D(t)C(t)\cos(\omega_c t + \theta_0)$$

这里: α_0: 衰减系数，对直达信号，α_0=1。

A: 直达信号的幅度

θ_0: 直达信号的载波相位

ω_c: 载波频率

$C(t)$: C/A 码

$D(t)$: 导航数据

在捕获环节和跟踪环节(码跟踪环和载波跟踪环)中，假设导航数据是不变化的，可以设 $D(t)$=1，此时直达信号的表达式可以简化为:

$$S(t) = A\alpha_0 C(t)\cos(\omega_c t + \theta_0)$$

复信号表达式:

$$S(t) = A\alpha_0 C(t)\mathrm{e}^{\mathrm{j}(\omega_c t + \theta_0)} \quad (1)$$

2.2 多径信号

如果在接收到的信号中存在 N 条多径，此时接收信号的表达式为:

$$S_{\mathrm{mp}}(t) = \sum_{i=0}^{N} A\alpha_i C(t-\tau_i)\mathrm{e}^{\mathrm{j}(\omega_c t + \theta_i)} + n(t) \quad (2)$$

其中: N 为多径的数目

α_i: 第 i 径反射信号的衰减系数(相对直达信号)且α_0=1

A: 直达信号的幅度

θ_i: 第 i 径反射信号的载波相位

ω_c: 载波频率

$C(t-\tau_i)$: 第 i 路延迟 C/A 码，而且 τ_0=0

$n(t)$: 均值为零的加性高斯白噪声(AWGN)。

由此可以看出，多径信号由反射信号的幅度、相位、延迟以及载波相位的变化决定。

2.3 本地载波和本地 C/A 码

本地载波由载波锁相环产生一对正交的载波，当载波环锁定时，其频率和接收到的信号完全一致，复信号表达式为:

本地载波: $F(t) = \mathrm{e}^{-\mathrm{j}(\omega_c t + \theta_c)}$

ω_c: 本地载波频率，同步时与接收到的载波频率相等

θ_c: 本地载波相位

本地 C/A 码: (早、晚、即时)

早码: $C_E(t)=C(t-\tau+\tau_d)$

晚码: $C_L(t)=C(t-\tau-\tau_d)$

即时码: $C_P(t)=C(t-\tau)$

其中: τ为 C/A 码从卫星到用户的传输时间的估计值，τ_d 为本地早/晚码与即时码的延迟，也就是早/晚相关器与即时相关器的间隔。

2.4 相关器的输出

GPS 信号采用扩频调制技术，导航数据首先被 C/A 码扩频调制，然后再对载波进行 BPSK 调制，因此，在 GPS 接收机中必须采用两个环路分别跟踪载波和 C/A 码。码跟踪环和载波跟踪需要相互配合才能正确跟踪到信号。载波跟踪环一般采用 Costas 环，因为 Costas 环对载波上的调制信号不敏感，对未解调的 GPS 信号是非常适合的。C/A 码跟踪环通常采用延迟锁相环(Delay Lock Loop DLL)，以实现对 C/A 码传输延时的最大似然估计(ML)。在码跟踪环中，有 6 个相关器，即在正交通道上有 3 个: 早相关

器、晚相关器和即时相关器。在同相通道上也有 3 个相关器: 早相关器、晚相关器和即时相关器。

接收到的信号首先分为两路, 分别被本地产生的两路正交载波相乘, 形成正交通道和同相通道的输入信号, 再分别与本地的早码、晚码、即时码分别相关。结果再送到码环鉴别器进行处理, 如图 1。

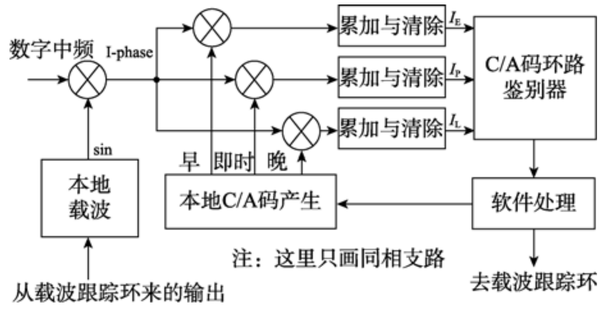

图 1　同相 I 通道的 DLL C/A 码跟踪环

实际上, 接收到的 GPS 中频信号, 与载波(同相和正交)相乘、再分别与 C/A 码的早、即时、晚码相乘, 然后经过积分与清除模块, 是为了实现相关运算, 其最后输出为: (早相关分量)

$$\begin{aligned}I_{\mathrm{E}} &= E\left\{\frac{1}{T}\int_0^T S_{\mathrm{mp}}(t)C_{\mathrm{E}}(t)\mathrm{e}^{-\mathrm{j}(\omega_{\mathrm{c}}t+\theta_{\mathrm{c}})}\mathrm{d}t\right\}\\ &= E\left\{\frac{1}{T}\int_0^T S_{\mathrm{mp}}(t)C(t-\tau+\tau_{\mathrm{d}})\mathrm{e}^{-\mathrm{j}(\omega_{\mathrm{c}}t+\theta_{\mathrm{c}})}\mathrm{d}t\right\}\\ &= E\left\{\frac{1}{T}\int_0^T \sum_{i=0}^{N} A\alpha_i C(t-\tau_i)\mathrm{e}^{\mathrm{j}(\omega_{\mathrm{c}}t+\theta_i)}C(t-\tau+\tau_{\mathrm{d}})\mathrm{e}^{-\mathrm{j}(\omega_{\mathrm{c}}t+\theta_{\mathrm{c}})}\mathrm{d}t\right\}\end{aligned}$$

结果:

$$I_{\mathrm{E}} = \sum_{i=0}^{N} A\alpha_i \overline{C(t-\tau_i)C(t-\tau+\tau_{\mathrm{d}})}\mathrm{e}^{\mathrm{j}(\theta_i-\theta_{\mathrm{c}})}$$

用 C/A 码相关函数来表示:

$$I_{\mathrm{E}} = \sum_{i=0}^{N} H_i R(\tau-\tau_{\mathrm{d}}-\tau_i) \tag{3}$$

其中:

$H_i = A\alpha_i\mathrm{e}^{\mathrm{j}(\theta_i-\theta_{\mathrm{c}})}$, $R(\tau)=\overline{C(t)C(t-\tau)}$ 为 C/A 码的自相关函数。

同理, 晚相关分量:

即:
$$I_{\mathrm{L}} = \sum_{i=0}^{N} H_i R(\tau+\tau_{\mathrm{d}}-\tau_i) \tag{4}$$

采用相干鉴相器, 则鉴相器的输出为:

$$\begin{aligned}D_I(\tau) &= I_{\mathrm{E}} - I_{\mathrm{L}}\\ &= \sum_{i=0}^{N} H_i[R(\tau-\tau_{\mathrm{d}}-\tau_i) - R(\tau+\tau_{\mathrm{d}}-\tau_i)] \qquad (5)\\ &= \sum_{i=0}^{N} H_i g(\tau-\tau_i)\end{aligned}$$

这里: $g(\tau)=R(\tau-\tau_{\mathrm{d}})-R(\tau+\tau_{\mathrm{d}})$

$R(\tau)$: 是 C/A 码的自相关函数。在没有多径时, C/A 码的自相关函数是标准的三角形; 当存在多径后, 三角形发生畸变变成了多边形如图 3、图 4 所示。

$g(\tau)=R(\tau-\tau_{\mathrm{d}})-R(\tau+\tau_{\mathrm{d}})$ 就是不存在多径时, 理想条件下的鉴相曲线, 通常称为 S-曲线。C/A 码的理想 S-曲线如图 2。

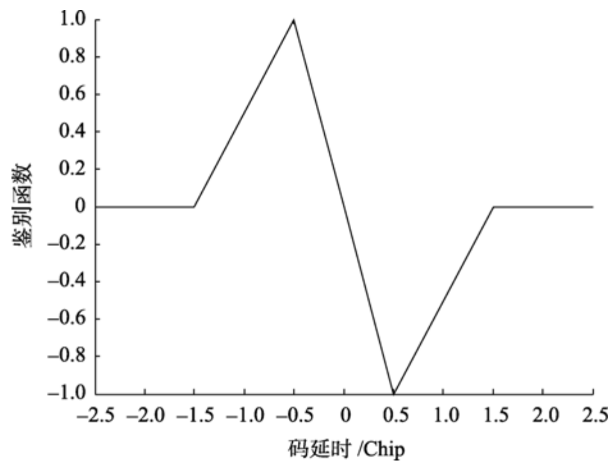

图 2　鉴相器归一化后的 S-曲线 (理想情况)

(S-曲线: $E-L=1T_{\mathrm{c}}$)

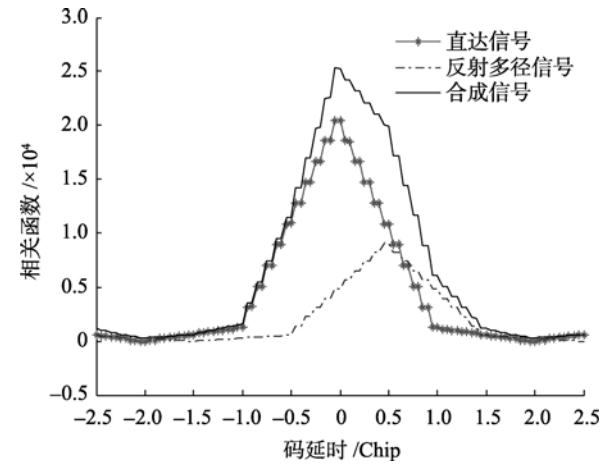

图 3　一条多径且相位与直达信号同相时的自相关函数

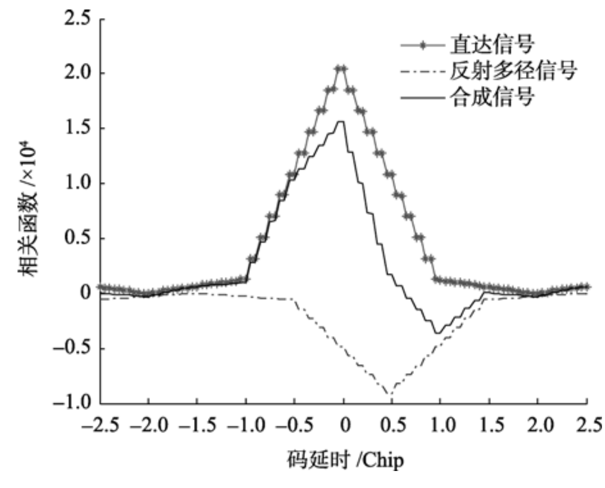

图 4　一条多径且相位与直达信号反相时的自相关函数

3 多径信道的冲激响应

根据公式(5):

$$D_I(\tau)=I_{\rm E}-I_{\rm L}=\sum_{i=0}^{N}H_i g(\tau-\tau_i)$$

选择足够小的Δ，使得τ和$\tau_{\rm d}$能够满足分辨率的要求。

$\tau=m\Delta$为待测的传输延迟，$\tau_i=i\Delta$为各路多径的延迟，最后鉴相器的输出可以表示成:

$$y(m)=\sum_i h(i)g(m-i)=h(m)*g(m) \tag{6}$$

这里: m为鉴相器输出数据的延时范围，

i为多径的数目。

其中: *表示卷积运算，

$$y(m)=D_I(m\Delta),$$
$$h(m)=H_m,$$
$$g(m-i)=g[(m-i)\Delta]$$

式(6)表明，当离散化以后，码跟踪环鉴相器的输出，是$h(m)$与$g(m)$的卷积。$h(m)$可以看成从GPS信号发送端到接收机这段衰落信道的系统冲激响应，$g(m)$为该系统的输入，输出为$y(m)$。衰落信道的冲击响应$h(m)$，反应了信道的多径效应，该信道的模型如图5所示。

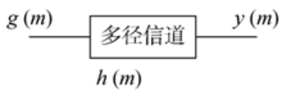

图5 信道模型

结论：只要能够估计出多径信道的冲激响应$h(m)$，就掌握了多径信道的特性，从而可以采用均衡器、逆滤波等技术，消除多径的影响。

4 自适应多径消除

根据前面的讨论，问题的关键是能够依据某种原则，估计出多径信道的冲激响应$h(n)$。有很多技术可以用于系统的估计，例如盲系统估计(Tugnait & Luo, 2002)、反卷积系统估计技术(Mohammad, 2005)等。这些技术，在估计出信道的冲激响应$h(n)$后，还必须从中将多径的影响部分剔除掉，如Rlinami(2000)，采用将$h(n)$中的第1个峰值作为直达信号，将其他较小幅度的$h(n)$作为多径剔除；Mohammad(2005)采用在$h(n)$之后再进行反卷积的方法消除多径的影响。

本文采用自适应滤波器，直接进行逆滤波，不需要先估计出$h(n)$，而是直接将多径滤除掉，恢复出原来的鉴相函数，提高码跟踪环的跟踪精度。如图6。可以看出，虽然没有估计$h(n)$，但是，却在进行逆滤波的过程中隐含地估计了$h(n)$。

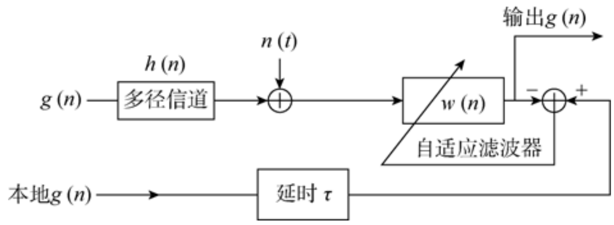

图6 自适应滤波器实现逆滤波

4.1 算法原理

在自适应滤波器中，参考信号的获取是一个重要问题。对于GPS，由于码跟踪环的鉴相函数$g(n)$是已知的，在理想情况下是双三角形(S 形状)，因此，自适应滤波的参考信号可以用本地的C/A码形成S-曲线，自适应滤波的输入为叠加有噪声的接收信号，利用RLS算法完成自适应滤波。实际上RLS算法采用的是最小二乘原理，在样本空间有限时，属于最佳估计(Haykin, 2002)。

假设: GPS信号的传输延时为$\tau_{\rm s}$，信道的冲激响应为$h(n)$，接收机码跟踪环DLL的鉴相函数为$g(n)$，则自适应滤波器的输入:

$$x(n)=g(n-\tau_{\rm s})h(n)+n(t)$$

自适应滤波器的参考信号为本地的鉴相函数:

$$d(n)=g(n-\hat{\tau}_{\rm s})$$

$\hat{\tau}_{\rm s}$为传输延时的本地估值。

经过滤波器后，得到的鉴相函数估计值:

$$\hat{g}(n)=x(n)*w(n)=[g(n-\tau_{\rm s})*h(n)+n(t)]*w(n)$$

自适应滤波器的估计误差:

$$e(n)=d(n)-\hat{g}(n)=g(n-\hat{\tau}_{\rm s})-\hat{g}(n)=g(n-\hat{\tau}_{\rm s})-x(n)*w(n) \tag{7}$$

误差的平方和:

$$\varepsilon(n)=\sum_{i=0}^{n}e^2(i)\to\min \tag{8}$$

在式(7)、式(8)中，$x(n)=g(n-\tau_{\rm s})*h(n)+n(t)$是接收机收到的信号，是已知的，$g(n-\hat{\tau}_{\rm s})$是本地的鉴相函数也是已知的，根据使平方和$\varepsilon(n)$最小的原则，就可以求得最佳的权值$w(n)$，这里采用自适应滤波算法。当自适应滤波器收敛时，获得的滤波器权值$w(n)$

就是式(8)的最小二乘解。

此时自适应滤波器的输出有:

$$\hat{g}(n) \approx g(n-\hat{\tau}_s)$$

而 $\hat{g}(n) = [g(n-\tau_s) * h(n) + n(t)] * w(n)$

$$\approx g(n-\tau_s) * h(n) * w(n)$$

$$\approx g(n-\hat{\tau}_s)$$

因此 $h(n) * w(n) \approx \delta(n)$

其 z 变换: $H(z)W(z)\approx 1$

或者: $W(z) \approx \dfrac{1}{H(z)}$

所以, $w(n)$可以看成是信道 $h(n)$在最小二乘意义上的最佳逆滤波器, 输入 $g(n)$经过多径信道 $H(z)$和级连的逆滤波器 $W(z)$后, 输出为:

$$\hat{g}(n) \approx g(n-\tau_s) * h(n) * w(n) \approx g(n-\tau_s)$$

也就是说, $\hat{g}(n)$ 就是 $g(n-\tau_s)$的最小二乘估值, 可以将多径 $h(n)$的影响剔除。

4.2 传输延时估计

码跟踪环的主要目的是, 精确的获得 GPS 的 PRN 信号从卫星到用户的传输延时的估计值进行伪距测量。

在式(7)、式(8)中, 给定 $\hat{\tau}_s$, 就可以求得自适应滤波器的最佳权值 $w(n)$, 因此, $w(n)$可以认为是 $\hat{\tau}_s$ 的函数, 误差的平方和 $\varepsilon(n)$也是 $\hat{\tau}_s$ 的函数, 在 $\hat{\tau}_s$ 的所有值域范围内,

使 $\underset{\hat{\tau}_s}{\|\varepsilon(n)\|} = \min$

则对应的 $\hat{\tau}_s$ 就是所求传输延时的最佳估计。

5 仿真结果

5.1 S 曲线修正

在码跟踪环 DLL 中, 环路锁定时, 其锁定点(平衡点)在 $\tau=0$ 处, 即锁定在鉴相函数(S-曲线)的过零点。当存在多径时, S-曲线是直达信号和各个多径信号的合成(叠加), 如式(6), 合成后的 S-曲线将发生畸变, 如图 7 的点划线。其过 0 点会发生偏移, 如图 7。

其中, 点划线为 $\alpha_1=-3$dB(0.5), 码延时 $\tau=1.1$ Chip, 载波相位 $\theta_1=0.15\pi$, 早—迟相关器间隔 $\tau_d=1.0$ Chip 的 S-曲线仿真结果。

从图 7 的仿真结果看出, 采用自适应滤波算法, 可以将受到多径污染发生畸变的 S-曲线矫正到接近理想的位置。过零点的偏移大为减少, DLL 的跟踪精度得到提高。

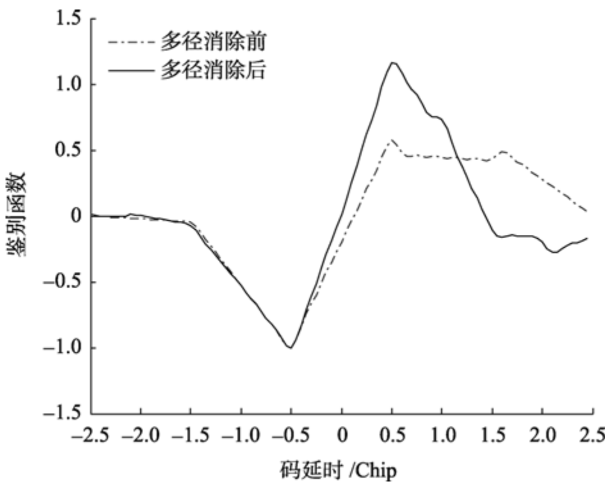

图 7 存在多径时的 S-曲线

(S-曲线 $E-L=1T_c$, BW=10.23MHz)

5.2 多径误差包络

当存在多径效应时, 合成信号的 S-曲线过 0 点与单独由直达信号形成的 S-曲线过 0 点就存在差异, 由于 DLL 会跟踪锁定在合成信号形成的 S-曲线过 0 点, 因此, 这个差值就是多径误差。将不同延时下的多径合成信号造成的多径误差画成曲线, 就得到误差包络曲线如图 8。

当多径信号与直达信号同相时, 其多径误差是正值, 形成误差包络的上半部分, 当多径信号与直达信号反相时, 其多径误差是负值, 形成误差包络的下半部分。图 8 为采用自适应多径消除算法前后的误差包络仿真结果。

从图 8 的仿真结果可以看出: 经过多径消除算法处理的误差包络比处理前的误差包络, 其幅度大为减小, 说明因多径引起的误差被大大衰减, 从而消除了多径的影响。尤其值得注意的是, 对于 $\tau<1.8$ Chip 的短延时的多径, 算法消除效果显著。

5.3 信噪比 SNR 和系统带宽 BW 的影响

图 8 为 SNR=10dB, 系统带宽 BW=10MHz 的情况下的误差包络仿真结果。

图 9 为 SNR=10dB, 系统带宽 BW=2MHz 的情况下的误差包络仿真结果。从结果可以发现, 算法对系统带宽不敏感, 在最小的系统带宽 BW=2.046MHz 的条件下, 多径消除效果依然很好。

图 10 为 SNR= −10dB, 系统带宽 BW=2MHz 的情况下的误差包络仿真结果。

从仿真的结果看, 当信噪比很低时(SNR<−15dB), 自适应滤波器很难收敛, 但是, 信噪比 SNR>

−10dB 以后，自适应滤波器迅速收敛，信噪比和系统带宽对算法的影响很小，如图 8—图 10。

这是因为，在实现逆滤波的时候，如果采用解析的方法(如：最小二乘法)，必然会涉及相关矩阵求逆的问题，由于噪声的增加导致奇异的相关矩阵，其逆矩阵就不存在。在自适应滤波器的情形，就会形成滤波器发散。

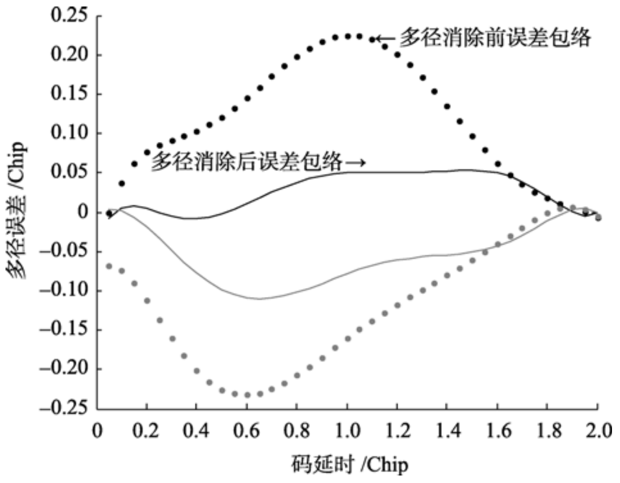

图 8 SNR=10dB, BW=10MHz 误差包络曲线

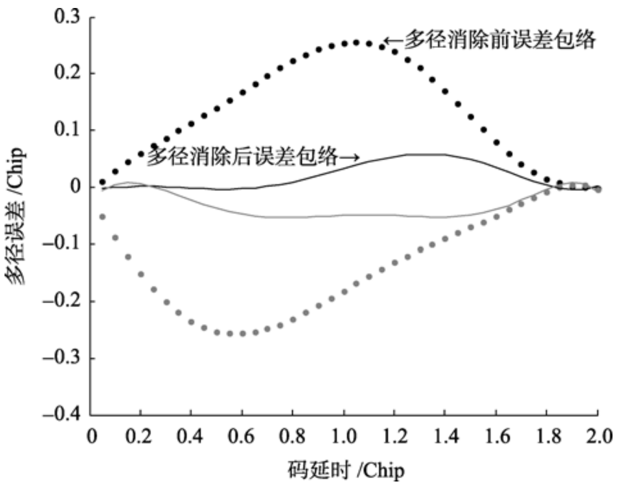

图 9 SNR=10dB, BW=2MHz 误差包络曲线

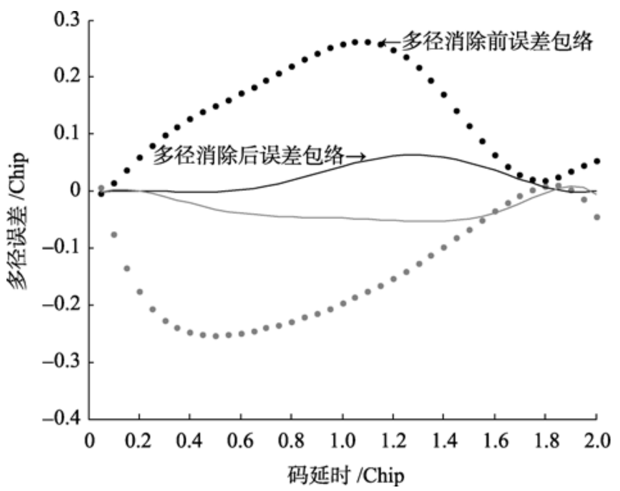

图 10 SNR= −10dB, BW=2MHz 误差包络曲线

实际上，本算法是在捕获之后进行的，GPS 信号经过捕获过程，信噪比已经大为提高，信噪比 SNR > −10dB 是很容易满足的。

6 结 论

本文在对 GPS 接收机码跟踪环的鉴相函数 S-曲线进行充分研究基础的上，提出了采用自适应滤波的算法来实现对多径信号的剔除，直接进行逆滤波，不需要事先估计出系统传输函数，直接将多径滤除掉，恢复出原来的鉴相函数，提高了码跟踪环的跟踪精度，从而消除多径对鉴相函数的影响。

在类似室内的定位应用即存在短延时多径干扰的环境下，窄相关器技术、选通相关器、边沿相关器技术等由于对短延时多径的消除效果降低，因此本文提出的自适应滤波的算法具有一定的优势。

仿真的结果表明，采用自适应滤波器算法可以快速的消除多径影响，使 S-曲线的过 0 点偏差减小、DLL 的跟踪精度得到提高，尤其是对短延时多径干扰的消除效果显著。算法对系统带宽不敏感，在中频最小带宽为 2.046MHz 的条件下多径消除效果依然很显著，在信号捕获后的信噪比为−10dB 的强干扰下算法仍然收敛得很快，并且由于自适应滤波算法是递推算法，易于软、硬件实现。

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