A novel SAR imaging method under geographical coordinates

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Abstract: Traditional airborne SAR (Synthetic Aperture Radar) makes use of the GPS/INS to compensate the platform motion error to acquire high resolution images. GPS data with geographical coordinates (consists of longitude, latitude and height) is converted to a local Cartesian coordinates, under which raw radar data is processed with some imaging and motion compensation algorithms to create an image with the local coordinates. However, it is not easy to be understood for other departments who need an image located by geographical coordinates. This paper presents a novel SAR imaging algorithm derived from Back-Projection that can be processed directly under the geographical coordinates and avoid coordinates transformation. Each pixel of the image obtained via this algorithm is located with geographical coordinates, which is convenient to be applied by the intelligence or other survey departments. Images obtained under geographical coordinates by this algorithm are as clear as traditional algorithms that been proved by simulations and an outside experiment.

Key words: Synthetic Aperture Radar, motion compensation, geographical coordinates, coordinates transformation, elevation calibration

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1 INTRODUCTION

Nowadays, Synthetic Aperture Radar (SAR) images are both used in military field and in other civil life such as nature disaster forecasting, geologic reconnaissance, geologic charting, polar region surveillance, forest exploitation programming (Luojus, et al., 2009; Zhang, et al., 2009). However, the images obtained by common SAR algorithms can not be applied directly as they are not located with global position information, in other words, there are variations between the image and ground coordinates. The radiometric slope of the image brought by the spherical fluctuating ground surface should be modified firstly (Ulander, 1996). The geocoding procedure is following, by which each pixel will get a pair of geographical coordinates (He & Wang, 2006). Thereafter, any targets interested in the SAR image could be located by their longitudes and latitudes (Yu, et al., 2007).

Airborne SAR makes use of GPS/INS to get real-time position information of the platform to compensate the motion error that is presented (Kirk, 1975). RTK (Real-Time Kinematic) GPS could produce position information whose error is less than 3 cm while updating rate to 50 Hz (Araki & Shikada, 2001; Earle, et al., 2005; Yang & Lo, 2000). Due to its high updating rate and no authorization restriction, it could replace GPS/INS system in some missions when the task scope is not too large as to overrun the RTK’s constraint. The UAV (Unmanned Aircraft Vehicle) SAR, has small observation area, small activity scope and it is appropriated to be equipped RTK GPS. Traditional SAR imaging algorithms and motion compensation algorithms are presented under Cartesian coordinates, accordingly, the geographical coordinates produced by RTK GPS should be transformed to Cartesian coordinates in advance. Finally, we can obtain an image under the local Cartesian coordinates and the targets could be relocated with the geographical coordinates.

There are two defects about traditional SAR algorithms. Firstly, traditional transformation between coordinates needs precise absolute coordinates. RTK GPS needs control points to calibrate the base station, only by which precise absolute coordinates could be obtained. But it is difficult to acquire the control points sometimes. Secondly, the two transform procedures will complicate the SAR processing procedure and introduce new errors. We are looking forward to finding a way to imaging directly under the geographical coordinates and there are seldom references we could find that mention any algorithm for imaging in this way.

This paper presents a novel imaging algorithm that developed from the Back-Projection (BP) algorithm (Rau & McClellan, 2000).
In this algorithm the RTK GPS data is introduced directly into imaging process, and then an image under geographical coordinates is obtained, avoiding the two transform procedures. The relationship between this algorithm and traditional algorithm is shown in Fig. 1. In this paper the radar is supposed to work on the strip-map SAR mode and the track of the airborne radar is supposed as linear with some intense vibrations in the slant range direction. The UAV’s observation area is relatively small so that the spherical ground surface could be approximated to a plane and the radio slope correcting is not needed here. The RTK GPS could only supply the relative coordinates without any control points. The reason for this algorithm can acquire an image with high quality is because only relative distance is needed.

2 MOTION COMPENSATION

Motion compensation is the key step in an airborne SAR imaging procedure. Time domain algorithm could realize this just as the BP algorithm (Rau and McClellan, 2000), which could be presented by

\[
f(x, r) = \int \int \frac{f(\tilde{x}, \tilde{t})w(\tilde{k}, \tilde{\ell}, \tilde{t})}{\tilde{r}} \left| \frac{d}{h} \right| \left| \frac{2R(x, r, x)}{c} \right| dt \, dx \, dy \, dz
\]  

where \( x, r \) denote the azimuth and range coordinates of the image pixel; \( f(x, r) \) is the image pixel value; \( t \) is the fast time; \( \tilde{x} \) is radar position corresponding to each echo column; \( \tilde{d}(\tilde{x}, \tilde{t}) \) is the echo matrix which has been range compressed, \( w(\cdot) \) is antenna shape function; \( d(\cdot) \) is the unit impulse function; \( R(x, r, \tilde{x}) \) is the distance between image pixel \( x, r \) and radar position \( \tilde{x} \). Note that the position of radar and image will both be denoted by geographical coordinates as \( b, l, h \). The radar position \( (b_0, l_0, h_0) \), which is offered by the RTK GPS, denotes the place where it acquires the \( k_0 \) column echo. \( (b, l, h) \) is the image pixel position \( p \). So Eq. (1) could be written as

\[
f(b', l', h') = \sum_{k} \frac{1}{R_b(b', l', h')} \frac{d_k(t)}{c} \left( t - \frac{2R_b(b', l', h')}{c} \right) dt
\]  

where \( w(k, p) = \left( \frac{\sqrt{R_k(p) - R_k(p')}}{R_k(p')} \right) R_k(p) \) is the nearest distance between the track line and pixel \( p' \). It is convenient to obtain the whole image by Eq. (3) when \( R_b(p) \) and \( R_b(p') \) are both known for each \( p' \). \( R_b(p') \) varies when the radar is moving, and it denotes the distance between point \( p' \) and the radar position when it transmitting the \( k_0 \) microwave. The motion features are included in \( R_b(p) \) as well, which can compensate the motion error.

3 IMAGING UNDER GEOGRAPHIC COORDINATES

There are many methods to calculate the distance \( R(k, p') \) between two points using their geographic coordinates (Sansosti, et al. 1997). One of them is presented as Eq. (4):

\[
f_{\theta}(p_1, p_2) = \sqrt{(l_2 - l_1) c \cos b_2} + (b_2 - b_1) c \sin b_2
\]  

where \( R_{\theta} \) is the polar radius; \( R_{\varphi} \) is the equatorial radius of the earth; \( (b_1, l_1) \) is the latitude and the longitude of point \( p_1 \), and \( (b_2, l_2) \) denotes the other point. The place in this study is near (N26°, E112°), so we set the beginning point as \( (b_1, l_1) \), where \( b_1 = 26° \), \( l_1 = 112° \), then calculate the distance variation with the longitude and the altitude changing. The result is shown in Fig. 2, x axis shows \( \Delta \theta \) and \( \Delta \lambda \) that are the variation of altitude and longitude, respectively, y axis shows the distance as \( R(\Delta \theta) = f_{\varphi}(b_2, l_2, l_1, \Delta \lambda) \) and \( R(\Delta \lambda) = f_{\theta}(b_2, l_2, l_1, \Delta \lambda) \). \( R(\Delta \theta) \) and \( R(\Delta \lambda) \) are illustrated separately by dots. Linear lines that almost superpose \( R(\Delta \theta) \) and \( R(\Delta \lambda) \) are drawn by lines. With this, it is easy to find that \( R(\Delta \theta) \) and \( R(\Delta \lambda) \) are very approximate to the linear functions within 100 km. Besides, the longitude lines are perpendicular to the latitude lines. Therefore, in our task the longitude latitude grid could be supposed to a special rectangular coordinates system. The axes are the longitude and latitude that have been divided into equidistant units separately with different scales. The height of the aircraft is supposed to be a constant during the flight, the ideal track of the airplane is expressed by a linear equation \( b = k_1 l + k_2 \), where \( b \) and \( l \) are the latitude and longitude of the airplane, \( k_1 \) and \( k_2 \) are the coefficient to be sought by Least Squares Method (LSM) through the actual points \( (b_2, l_2) \) known by the RTK system according to Eq. (5). As shown in Fig. 3, line \( a_1 \) is the actual track, \( a_2 \) denotes the ideal track.

\[
\sum_k (k_1 l_k + k_2 - b_k)^2 \rightarrow \min
\]  

We can define one axis of the coordinates as the flight direction under the local Cartesian coordinates easily, then the observation area becomes a strip region along the track. The area could be expressed by \( \Delta x_{\text{min}} \) and \( \Delta y_{\text{max}} \). However, it is difficult to do the same work under the geographic coordinates because...
the flight track will not always be paralleled to either longitude or latitude lines. To find a way to express the observation area is one important step of this algorithm. A matrix \( Q_{\xi \eta} \) whose cell \( (b_{\xi \eta}, L_{\xi \eta}) \) is of geographic coordinates is needed to denote the imaging area.

The imaging process should focus on the interested area rather than a great deal of trashy area, by which the imaging efficiency will increase substantially. Firstly, determine the irradiated area’s minimum distance \( d_i \) and maximum distance \( d_o \) away from the track’s projection on the ground, and the area’s length \( d_l \) along cross range direction. These parameters can be obtained by the known beam angle, the slant angle and the flight height. Define the mean value of the serial track coordinates as \( \bar{l} \) and \( \bar{b} \), and calculate \( \Delta l \) that is the distance between two points near \( (\bar{l}, \bar{b}) \) with 1° longitude variation. Use the same method to calculate \( \Delta b \). Follow this, calculate the position \( D \) \( (b_L, l_D) \) by \( d_i \) and the track coordinates as shown in Fig. 3. A dashed frame whose one vertex is \( D \) is shown in Fig. 3, and its longitude range is \( [l_D-l_0,l_D+l_0] \), latitude range is \( [b_L-b_0,b_L+b_0] \). It is possible to divide the rectangular \( D \) into a grid \( (b_{L\xi}, L_{\xi}) \) by the aimed resolution \( \Delta l \) and \( \Delta b \). Then the grid \( (b_{L\xi}, L_{\xi}) \) could be acquired by shifting and rotating the coordinates according to

\[
\begin{align*}
\Delta l_{\xi} &= (b_{\xi,0} - b_0) \cos \theta + (L_{\eta,0} - l_0) \sin \theta + b_D \\
\Delta L_{\eta} &= -(b_{\xi,0} - b_0) \sin \theta + (L_{\eta,0} - l_0) \cos \theta + l_D
\end{align*}
\]

(6)

where \( \theta \) is determined by the flight track, i.e. \( \theta = \arctan \frac{\Delta l}{\Delta b} \).

Given \( A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \),

\[
T_{m,n} = \begin{bmatrix} b_{L,m,n} \\ L_{\xi,m,n} \end{bmatrix} = T_{m,n} = \begin{bmatrix} b_D \\ l_D \end{bmatrix}
\]

\[
A \times (T_{m,n} - T_{\xi,0}) + T_{\eta,0} = T^\prime_{m,n}
\]

(7)

According to Eq.(3), the pixel value of each cell \( (b_{L\xi}, L_{\xi}) \) of \( Q_{\xi \eta} \) can be calculated one by one when the height of each point is supposed to a same value \( h_0 \). Obviously, the computation pressure is as heavy as the BP algorithm. The total operation count is \( OP = L \times M \times N \), where \( L \) is the aperture length, \( M \times N \) is final size of the image. So the dividing method introduced above, by which the imaging area becomes smaller, is necessary to increase the computing speed. Please note that \( R_i(p) \) should be calculated first to get the minimum value \( R_i(p') \).

\[
R_i(p') = \min_{p} \{ R(k, p') \}
\]

(8)

A hypothesis in the forward process is that the area to be imaged is of the same height, which means the ground could be approximated to an ideal plane. It will be illconsidered in a mountainous region or a ground with intense height variation (Kwok, et al., 1987; Sansosti, et al., 1997), and the migration between image and ground coordinates should be corrected (Caves, et al., 1991; Hong, et al., 2004; Noack, et al., 1987). If the Digital Elevation Model (DEM) of the imaging area is known in advance, it is easy to acquire the image by replacing \( h_i \) with \( h = g(h_i) \) according to Eq.(3). If the DEM, flight parameters and the SAR image with geographical coordinates are known except for raw echo data, an additional step to correct the image called elevation calibration will be needed. This procedure is presented as following: point \( A \)’s actual geographic coordinates \( (b_{\xi A}, L_{\eta A}, h_i) \) is known and its coordinates \( (b_{\xi A}', L_{\eta A}', h') \) in SAR image is to be solved. After that the pixel value of \( A \) could be obtained by interpolating the known image. It is actually a projection procedure from true topographic data to SAR image. The SAR image only has region and azimuth information (Eineder & Adam, 2005), so the projection is feasible because it is from high dimension to low dimension.

Fig. 4(a) is the view along the flight direction, and Fig. 4 (b) is the view from sky to ground. In Fig. 4(a), the dashed line denotes the ground surface, \( O \) is the intersection of the paper plane and the flight track that is perpendicular to the paper plane, the horizontal axis denotes the plane where the SAR image is located at, the vertical axis denotes the height. For an actual point \( A \) with a \( h_i \) vertical distance from the image height, its projection to the image will be \( A' \). Point \( O \) is equidistant from \( A \) and \( A' \), and the location error is \( \Delta R_1 \) that is made use of to correct the geographic coordinates. It is obvious that

\[
\Delta R_1 = \sqrt{R_1^2 - h_0^2} \approx (h_0 - h_i)^2
\]

(9)

\[
\begin{align*}
\Delta l_1 &= \Delta L_1 = \Delta L_2 = \cos \theta \Delta \xi \\
\Delta b_1 &= \Delta b_2 = \Delta b_3 = \sin \theta \Delta \eta
\end{align*}
\]

(10)

where \( \Delta L_1 \) and \( \Delta L_2 \) are the coordinates corrections that from \( b_{\eta,0} \),
l₁, h₁) to \((b', l', h')\), \(\theta'\) denotes the flight direction under the coordinates whose axis steps at a fixed distance, yet \(\theta\) defined above is under the coordinates whose axis steps at a fixed longitude or latitude. \(\cos\theta'\) and \(\sin\theta'\) in (10) could be solved by

\[
\begin{align*}
\cos\theta' &= \frac{R_x}{R} f_x(p_x, p_{x0}) + f_x(p_{y0}, p_{y0}) \\
\sin\theta' &= \frac{R_y}{R} f_y(p_x, p_{x0}) + f_y(p_{y0}, p_{y0})
\end{align*}
\]

(11)

where \(p_x=(b, l)\) is mean value of the track coordinates sequence, \(p_{x0}=(b_x, l_x)\) is start point of the track, \(p_{x0}=(b_y, l_y)\) is end point of the track, and \(p_{x0}=(b, l), p_{x}= (b', l'), p_{y}=(b', l), p_{y0}=(b, l)\). As shown in Fig. 4(a), the point B is \(h_2\) below the imaging plane, so the \(h_2\) will be minus causing \(\Delta R_x\) minus accordingly. If the DEM data is of low resolution, we should interpolate it to high resolution first. After processing all the points in the true geographical map, we can obtain a corrected and geocoded SAR image. It should be noted that, the whole imaging process does not need any absolute coordinates, and it only needs precise relative coordinates. It is known that RTK method presented in this paper will be accomplished. Take the bottom target for example, it is located at \((28.00201°, 111.99899°)\), and this range minus 28° forms the final range. In this experiment, certain areas are useless and the nonirradiated areas can lead to a waste of computation. Actually the valid observation area is the region inside the dashed rectangle in Fig. 5(a). If we divide the imaging area along the flight track according to the steps presented in this paper, we could get the valid rectangular area, which is shown in Fig. 5(b). In this image, the axis does not denote the longitude or latitude anymore, because none rows of the pixels share the same latitude and none columns of the pixels share the same longitude. Therefore, the length and width of the image could be scaled, with each pixel having separate geographic coordinates. There will not be useless area imaged in Fig. 5(b), and the computation efficiency can be increased. Then the ground surface is supposed fluctuating by setting the targets’ altitude with 1—2 m variations, and the common imaging result is shown in Fig. 5(c). The “T” has been distorted because of the variations in targets’ altitude. It has been found that the true position of the target may no longer be a highlight point in the image. Now the elevation calibration method presented in this paper will be accomplished. Take the bottom target for example, it is located at \((28.00201°, 111.99899°)\) and \(-2\) m below the original image altitude. Calculate Eq.(9) to get \(\Delta R_x=-0.43668\), and get \(\sin\theta'=-0.4749, \cos\theta'=0.6638\) by Eq. (11). Then the corrected coordinates of the target in the image is obtained as \((28.00212°, 111.99898°)\), which is the exact position of the bottom highlighted point in the image. If we know all the altitudes of points, a corrected and geocoded SAR image will be acquired.

In the outside experiment, an impulse radar is set about 4 m high on a vehicle radiating side of the forward direction. This vehicle is moving along a curve approximating to a straight line to simulate the airplane’s flying condition. Five small trihedral angles are set as “+” on the flat ground as shown in Fig. 6(d). Due to the flat ground, the elevation calibration is not necessary. An area of about 4 m×5 m including these trihedral angles is to be imaged, and the result is shown in Fig. 6(a). We use traditional back-projection.
algorithm under local Cartesian coordinates to form an image that is shown in Fig. 6(b). Obviously, the coordinate transformation is inevitable, but there is no prior known control point. So the error introduced by the coordinate transformation causes the low precision of imaging. Reconnaissance by UAV for some special kind of targets could produce a series of images like Fig. 6(a), and then make use of them to form reliable map information. Take the map within [E112°51'58.615", E112°51'58.916"] and [N28°13'01.873", N28°13'02.533"] for example, we could detect the targets’ pixels first and put them on the map by its geographical coordinates which is shown in Fig. 6(c). Reconnaissance could be accomplished by a multi-mission for each mission finishing a part of the map. Then an exhaustive map containing these special targets will be accomplished.

<table>
<thead>
<tr>
<th>Target</th>
<th>New algorithm</th>
<th>Traditional algorithm</th>
<th>Distance /m</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>51°58.7946</td>
<td>51°58.7952</td>
<td>0.1129</td>
</tr>
<tr>
<td>②</td>
<td>51°58.8370</td>
<td>51°58.8371</td>
<td>0.0678</td>
</tr>
<tr>
<td>③</td>
<td>51°58.8000</td>
<td>51°58.8000</td>
<td>0.0554</td>
</tr>
<tr>
<td>④</td>
<td>51°58.7622</td>
<td>51°58.7622</td>
<td>0.0800</td>
</tr>
<tr>
<td>⑤</td>
<td>51°58.8022</td>
<td>51°58.8034</td>
<td>0.0644</td>
</tr>
</tbody>
</table>

Table 1 Targets coordinates

Fig. 5 “T” targets imaging results
(a) divide imaging area according to the longitudes and latitudes; (b) divide the imaging area according to the steps presented in this paper;
(c) heights variations supposed among the targets

Fig. 6 The imaging results of an experimental data
In order to compare the location precisions between the new algorithm and the traditional algorithm, the targets coordinates located by both two algorithms are shown in Table 1. Targets’ distances between positions located by different algorithms of the same target are also included in Table 1. Though we do not have the true absolute coordinates of the targets, distance between target ① and target ③ is set as 1 meter before. The distance of target ④, ③, ⑤ are also set 1 m. These distances are calculated and are shown in Table 2, from which we can find that the new algorithm produces better results than the traditional algorithm. The error of the distance is near 3 cm which is about the utmost measurable precision of RTK GPS. So the algorithm presented in this paper does not increase the location error.

<table>
<thead>
<tr>
<th>Targets</th>
<th>New Algorithm /m</th>
<th>Traditional Algorithm /m</th>
<th>True distance /m</th>
<th>Whose Error is small</th>
</tr>
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<tr>
<td>①③</td>
<td>1.0082</td>
<td>0.9511</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
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<td>1.0126</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>④③</td>
<td>1.0334</td>
<td>1.0354</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
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<td>1.0146</td>
<td>1.0146</td>
<td>1</td>
<td>N/T</td>
</tr>
</tbody>
</table>

5 CONCLUSION

A novel SAR imaging algorithm that introduces the RTK GPS data to compensate the platform motion error and to geocode the image under geographical coordinates is presented in this paper. This algorithm avoids additional coordinate transformation, which will simplify the procedure of airborne SAR reconnaissance. Both imaging area dividing and elevation calibration methods are presented, which are demonstrated to be inevitable in an airborne SAR reconnaissance. It is worthy of remarking that a plane approximation is introduced for geographical coordinates, hence, avoiding the imaging area being very large. However, this constraint is weaker than the constraint of RTK GPS system which allows a largest working in a scope of only 20 km. If RTK GPS works on a large range mode via GPRS net, the measurement error will be increased working in a scope of only 20 km. If RTK GPS works on a large than the constraint of RTK GPS system which allows a largest.

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后向投影算法的地理坐标系下的合成孔径雷达成像

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摘 要: 传统的机载合成孔径雷达(SAR)一般依赖于GPS/INS组合系统补偿平台的运动误差，并获得高分辨率的图像。GPS获取的地理坐标(即经度、纬度和高度)需要转换为本地的直角坐标，原始雷达数据在这个直角坐标系下才能引入成像及运动补偿算法进行处理，并获得本地直角坐标系下的图像。这种图像是通过局部坐标描述的，对于其他部门是不通的，因为其他部门需要的是经过全球地理坐标标绘的图像。本文提出了一种由后向投影算法BP(Back Projection)引出的新的SAR成像算法，它直接在地理坐标下处理，可避免坐标转换的过程。而且生成的图像像素均和地理坐标一一对应，能够很方便地被情报或其他部门使用。而且在仿真试验以及外场试验中，证明了本文算法和常规BP算法成像效果是相当的。

关键词: 合成孔径雷达，运动补偿，地理坐标系，坐标转换，高程校正

中图分类号: TP722.6
