The least square surface matching method for airborne LiDAR strip adjustment

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Abstract Based on Microsoft VS 2008 C++ platform, the least squares surface matching algorithm for airborne the LiDAR strip adjustment is realized. It refers to and improves Robert’s 3D surface matching algorithm by introducing the Gauss-Markoff model, acquiring the unbiased minimum variance estimation for the transformation parameters between adjacent strips. The real different data sets are used to validate the method. We study the need for using Gauss-Markoff model, efficiency and iterative convergence of the algorithm, the matching accuracy. The experimental results demonstrate that after correction the point clouds show much better alignment and the vertical matching error is less than 0.05 m for idea data, while poor quality data the error is slightly larger.

Key words: LiDAR, strip adjustment, LSM, Gauss-Markoff model

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1 INTRODUCTION

The LiDAR data is collected in strips. In applications, where a rectangular block or corridor is flown in a parallel line pattern, minimal overlap between neighboring strips is typically required to maintain contiguous coverage of the ground. Ideally, there should be no visible or measurable differences between overlapping the LiDAR strips, except for sensor noise, mainly caused by varying performance of the geo-referencing component. However, strip differences frequently occur. The differences between various LiDAR strips acquired over the same area is more visible in areas that is rich in objects of simpler geometric shapes, such as man-made objects, e.g., buildings, as shown in Fig.4 (a) and Fig.5 (a).

The primary objective of the LiDAR strip adjustment is to provide quality assurance and quality control (QA and QC) for the final geospatial product by reducing, or ultimately eliminating, discrepancies found in strip overlap areas, and thus create seamless product. The extent is significantly smaller than it was when the first generation of the commercial LiDAR systems was introduced. At that time, only the vertical accuracy of the LiDAR data was specified and no reference was provided for horizontal precision; consequently, the strip adjustment was aimed at removing only the height differences. Motivated primarily by generating DEMs, the first guideline to report on the LiDAR data quality was only concerned with the vertical accuracy. These strip adjustments can be referred as 1D strip adjustment methods (Crombaghs, et al., 2000; Kager & Kraus, 2001; Kornus & Ruiz, 2003; Tong, 2005).

With the advancement of the LiDAR technology, better point density measurements and point accuracy becoming possible, however, the changed situation and users started to recognize the true 3D nature of the LiDAR data. In particular, the importance of the horizontal component was widely acknowledged. Vosselman and Maas (2001) showed that systematic planimetric errors are often much more significant than vertical errors in LiDAR data and, therefore, a 3D strip adjustment is the desirable solution for minimizing the 3D discrepancies between overlapping strips and at control points.

The first 3D strip adjustment method developed was adjusted only for 3D offsets. Also, it was considered data driven techniques, as there was no attempt to rigorously model systematic sensor errors. In contrast, the second approach is concerned with the source of the errors and aims to reduce the strip discrepancies by modifying or adjusting the parameters of the physical sensor model and sensor orientation measurement system (the introduction of self-calibration), that is sensor calibration techniques (Behan, 2000; Favey, 2000; Burman, 2000, 2002; Morin & El-Shemy, 2002; Filin, 2001, 2003; Skaloud & Lichti, 2006; Liu, et al., 2005; Xu, 2006; Wang, et al., 2007; Zhang, 2009).

In this paper, a least squares surface matching algorithm is applied to the LiDAR strip adjustment. It refers to and improves Robert’s 3D surface matching algorithm by introducing the Gauss-Markoff model, acquiring the unbiased minimum variance estimation for the transformation parameters between different strips.
Two real different data sets are used to validate the method. We study the need for using Gauss-Markoff model (Li, 2001) and the matching accuracy. Profiles of specific features before and after the correction are created to show mismatch and match between different strips. In all experiments, after correction the point clouds show much better alignment; “Measure Match” tool of TerraMatch was used to measure how well flightlines match, “Average Magnitude” indicator is less than 0.05 m for ideal data, while poor quality data the error is slightly larger.

This paper is organized as follows. The details of the LSM algorithm and the key technology are explained in the following section. The matching procedures are given in the third section. Two practical examples for the demonstration of the method are presented in the fourth section. Summary and conclusion are given in the fifth section.

2 THE LEAST SQUARES SURFACE MATCHING THEORY

In this chapter, the least squares algorithm is described. The Eq.s used in the least squares solution are derived. Details of the program such as the establishment of the points correspondence, the iteration termination and the elimination of outliers are developed.

2.1 The least squares techniques

The model estimates the parameters of a seven-parameters conformal transformation which brings two surfaces to a position of minimal separation. A conformal transformation is one in which the relative relationship between the points of the transformed set is not changed.

The reference surface \( S_1 \) is triangulated. For each point of the surface \( S_1 \), the correspondence with \( S_1 \) is established (i.e., a triangle of \( S_1 \) is found) (see 2.2). It is assumed that \( S_1 \) can be transformed to a new position which minimizes the separation between the two surfaces. The separation is measured as the set of the normal distances from the points of \( S_1 \) to the corresponding facets of \( S_1 \). The parameters of the transformation are sought for by the LSM algorithm. \( S_1 \) is then transformed using these parameters. The process is iterative: the convergence is tested and the process is started again.

Let the three rotation angles be \( \omega, \phi, \kappa \), the three translations be \( T_x, T_y, T_z \), and the scaling factor \( s \) be the seven parameters, and then the normal distance from a point \( I' \in S_1 \) to a plane defined by the three vertices \((P, Q, R) \subset S_1 \) of the triangle enclosing the point \( I' \) is given by:

\[
D = |ax' + b(y' + z') + c|/\sqrt{a^2 + b^2 + c^2} \tag{1}
\]

where \( I' (x', y', z') \) is a function of \( I(x, y, z) \), \( \omega, \phi, \kappa, T_x, T_y, T_z \), and \( s \); \( a, b, c, d \) are functions of \( P(x_p, y_p, z_p), Q(x_q, y_q, z_q) \), and \( R(x_r, y_r, z_r) \).

An approximation \( D' \) of the normal distance \( D \) is given by linearising Eq.(2). Using a Taylor expansion:

\[
D' = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial s} \Delta s \tag{2}
\]

where \( D_0 \) is the distance \( D \) evaluated at the initial value of the parameters of the transformation, \( \Delta \omega, \Delta \phi, \Delta \kappa, \Delta T_x, \Delta T_y, \Delta T_z, \Delta s \) are corrections to initial values of the parameters.

Let \( V \) be the difference between the approximated normal distance \( D' \) and the exact distance \( D \):

\[
D = D' - V \tag{3}
\]

where \( V \) is due to higher order derivatives of the Taylor expansion, interpolation errors due to the difference between modeled and true reference surface and more generally sampling error. Eq.(3) can thus be written as:

\[
D = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial s} \Delta s - V \tag{4}
\]

With the model requirement that \( D = 0 \) (nil separation between the surfaces) and rearranging (4):

\[
V = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial s} \Delta s \tag{5}
\]

This calculation repeated over the \( n \) number of observations is arranged in matrix notation to give:

\[
V = L = AX \cdot P \tag{6}
\]

where \( A \) is design matrix, \( V \) is vector of residuals, \( L \) is vector of “observables” \( D_0 \) and \( X \) is vector of correction to the initial values of the parameters.

With the statistical expectation operator \( E[] \) and the assumptions:

\[
V \sim N(0, \sigma_1^2 Q_0), \sigma_2^2 Q_0 = \sigma^2 P_0^{-1} = K_1 = E[V'V] \tag{7}
\]

The Eq. (7) is a Gauss-Markoff estimation model. \( Q_2 \) and \( K_1 \) stand for a priori cofactor and covariance matrices, respectively. The unknown transformation parameters are treated as stochastic quantities using proper a priori weights. This extension gives advantages of control over the estimating parameters. We introduce the additional observation equation was introduced.

\[
V_I = IX + L_0 \cdot P_0 \tag{8}
\]

where \( I \) is the identity matrix, \( L_0 \) is the (fictitious) observation vector for the system parameters, and \( P_0 \) is the associated weight coefficient matrix. The weight matrix \( P_0 \) has to be chosen appropriately, considering a priori information of the parameters. An infinite weight value \( (P_0)_{ii} = \infty \) excludes the i-th parameter from the system assigning it as constant, whereas zero weight \( (P_0)_{ii} = 0 \) allows the i-th parameter to vary freely assigning it as free parameter in the classical meaning.

The least square solution of the joint system (6) and (8) gives as the generalized Gauss-Markoff model the unbiased minimum variance estimation for the parameters:

\[
\hat{X} = -(A' PA + P_0)^{-1} (A' PL + P_0 L_0) \tag{9}
\]

\[
\hat{V} = AX + L \quad \hat{V}_I = IX - L_0 \tag{10}
\]

\[
\hat{u} = (V' P_0 V + V'I_0 V')^{-1} \tag{11}
\]

where \( ^\prime \) stands for the least squares estimator, \( r = n - u \) is the redundancy, \( n \) is the number of observations that is equivalent to the number of elements of the template surface, and \( u \) is the number of transformation parameters that is seven here.
2.2 Point correspondence

The points of the two surfaces do not correspond. The correspondence is established by associating a patch of \( S_1 \) with a point of \( S_2 \). The programme associates the triangle enclosing the point: the vertical line passing through the point also passes through the triangle selected. As the surface is 2.5D, there can be only one triangle associated with each point. It can happen that the point of intersection of the normal and the plane is outside the triangle, depending on the separation and the slope of the triangle. The normal distance to the plane defined by the triangle is nevertheless calculated. This problem solves itself as the surfaces come together and the separation decreases.

2.3 Blunders and outliers

The outliers due to erroneous measurements and occlusions may significantly impair the quality of the registration. Blunders and outliers which could influence the solution is eliminated. The exclusion rule uses eliminates normal distances which differs from their mean by a value greater than three times standard deviation (i.e. the exclusion factor is 3). The programme allows the value of the exclusion factor to be changed. A smaller factor (e.g. 2) includes in the least squares solution only the distances whose differences are less than twice the standard deviation of the mean. Such a solution is useful when the LSM programme is involved in deformation measurement.

2.4 The discrepancies between two overlapping strips

Algorithm assumes that the orientation difference between two surfaces is small. Therefore, we need to provide initial approximations of the unknowns, to ensure convergence to the global optimal solution. For the LiDAR data, the situation is rather special, we just consider the initial value of the three translation parameters, because the three rotation parameters of the LiDAR data will not be too large, the initial value of the whole can be assigned 0, while the initial value of scaling factor can be assigned one. For this problem, a search-based approach is used. First, extract feature points with Moravec operator in the reference surface, and then move search surface with feature points as the center within a certain area, calculate the correlation coefficient, to search for the best approximate matching location.

2.5 Algorithm optimisation

Translations in easting, northing and height should be applied to the data to bring the coordinates of the points to a position where the centroid of the set is zero approximately. Effectively, geographical data often comes in many digits. For example the coordinates of (265732.80, 3995270.97, 63.87), these large numbers exacerbate numerical errors and numerical error propagation. In addition, the high correlation between the parameters of the transformations is removed by shifting the origin of the coordinate system toward the centroid of the set. Pre-processing consists of shifting the origin of the reference system close to the centroid of the reference set. The search surface is transformed using the same translations before matching is attempted. The sets are reverted back to their original coordinate system after matching. Using the centroid of the reference set as the origin of the matching coordinate system also simplifies the matching transformation, as the rotations have smaller lever arms.

3 THE MATCHING ALGORITHM PROCEDURE

3.1 Data pre-processing

As the matching procedure depends on continuous surfaces, the laser data has to be filtered to get ground points.

3.2 The initial value of the transformation parameters

In all experiments the discrepancies between two overlapping strips are provided by search-based approach (see 2.4); the scale factor is fixed to unity; the initial value of the three rotation parameters is fixed to 0.

3.3 The flowchart of basic surface matching

Several criteria can be used for the termination of the iterations (Fig. 2). Criterion 1: the maximum number of iterations
Dicates that the solution is converging.

4 THE EXPERIMENTAL RESULTS

The algorithm was tested on two real data sets. All experiments are carried out using own self-developed VC++ software that runs on Microsoft Windows XP.

4.1 The experimental data

The first data set: the sample data of Terrasolid software. The calibration flight is made by TopEye, these missions are flown in a four leaf pattern over an area with distinct feature on ground. The feature is a broad bank (Fig. 3(a)).

The second data set: ALS50-Ⅱ calibration data, but from the results of flight calibration we know that the attitude & position parameters calculated from integrated IMU/DGPS systems are not accurate enough, the relative offset between two strips reached to a maximum value of 23 m (Table 1). So, the data is poor quality. The feature is gable roof (Fig. 3(c)).

4.2 The experimental results

4.2.1 The first experiment

The experiment is designed to study the matching precision of different reference surface from two strips. Matching accuracy is measured by “Measure Match” tool of TerraMatch and expressed by “average magnitude” indicator (magnitude tells how much points of one flightline differencing on the average from the other flightlines. If the systematic vertical offsets are removed, magnitude tells how much noise on flightline).

The experimental results show that: the matching precision of different reference surface is different. In matching experiment of “1 and 4”, matching precision is 0.04521 m while strip 1 as reference surface; matching precision is 0.03646 while strip 4 as reference surface. This may be related to point cloud density of overlapping region, because the points of S1 and S2 are both sampled from the true surface. Theoretically, assuming that the two sets S1 and S2 are both free of sampling errors, the matching accuracy improves with increasing density of the reference surface. Following this theory, accuracy of matching and faithfulness of the reference surface to the true surface are synonymous and express the same concept. We can refer to Table 2 “1 and 4”, strip 4 point density is bigger and S2=4 is higher than S1=1. Comparison of Table 3 and Table 2, it is not difficult to find the level of precision consistent with the density. Therefore, in adjustment process we can directly set big density surface as the reference surface.

4.2.2 The second experiment

The experiment is designed to study the need for using Gauss-Markoff model. In Table 4, estimated parameters are \( T_x, T_y, T_z, s, \phi, \omega, \kappa \) in turn; \( P_b \) is priori weight coefficient matrix of corresponding parameters; \( P_r \) settings based on this consideration: airborne LiDAR data rotation parameters is not too large, the initial value and corresponding weight always assign 0, allowing parameters to vary freely; Then just study the remaining four parameters weight changes, if we think translation parameters and scale factor have the same weight, just consider 0, 10, 100, 1000, 10000 five cases, respectively.

<table>
<thead>
<tr>
<th>Table 1 List of experimental data sets</th>
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<tbody>
<tr>
<td>data set</td>
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<tr>
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</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<table>
<thead>
<tr>
<th>Table 2 Different reference surface matching experiment</th>
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<tr>
<td>strips</td>
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<tr>
<td>average magnitude /m</td>
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<table>
<thead>
<tr>
<th>Table 3 The number of ground points in overlapping region</th>
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<tbody>
<tr>
<td>strips</td>
</tr>
<tr>
<td>The number of ground points</td>
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</table>
Table 4 The weight matrix $P_b$ impact on the parameters

<table>
<thead>
<tr>
<th></th>
<th>$P_b$</th>
<th>estimated parameters</th>
<th>average magnitude/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0,0,0,0,0,0</td>
<td>1.0493,−1.3938,−0.7628, 0.99663,0.01053,−0.00283, −0.00120</td>
<td>0.02298</td>
<td></td>
</tr>
<tr>
<td>10,10,10,10,0,0</td>
<td>1.0239,−1.3822,−0.7479,0.99696,0.010422,−0.002850,−0.00134</td>
<td>0.02286</td>
<td></td>
</tr>
<tr>
<td>100,100,100,100,0,0</td>
<td>0.9823,−1.3802,−0.6585, 0.998058,0.00955,−0.00294,−0.00131</td>
<td>0.03186</td>
<td></td>
</tr>
<tr>
<td>1000,1000,1000,1000,0,0</td>
<td>1.0037,−1.5077,−0.7884, 0.99501,0.010481,−0.00280,−0.00209</td>
<td>0.02561</td>
<td></td>
</tr>
<tr>
<td>10000,10000,10000,10000,0,0,0</td>
<td>1.00006,−1.49902,−0.0589846, 0.999294,0.00225175,−0.00336645,−0.000637851</td>
<td>0.09401</td>
<td></td>
</tr>
</tbody>
</table>

Note: the initial transformation parameters ($1,−1.5,0,1,0,0,0$)

The experimental results show that: zero weight $(P_b)_{i,j} = 0)$ allows the $i$-th parameter to vary freely and estimated parameters have the maximum variation; with the increase of weight (0 to 10000) the variation of estimated parameters become smaller and smaller; when the weight is 10000, the corresponding parameters basically unchanged. Therefore we can control estimated parameters by changing the weights; “average magnitude” indicator indicates that we can improve the matching accuracy only by setting the appropriate weights. This also shows the need for using Gauss-Markoff model.

4.2.3 The third experiment

The Experiment is designed to study the accuracy of algorithm, as shown in Table 5.

The experimental results show that: (1) Overall, “average magnitude” indicator is less than 0.05m for ideal data (the first data set), while poor quality data (the second data set) the error is slightly larger. This may be related to poor data quality and inconsistency of local deformation. (2) Only from the perspective of “average magnitude” indicator, the accuracy of our algorithm is slightly lower than TerraMatch software. But in all experiments our LSM algorithm successfully completed the strip adjustment, after correction for the estimated parameters the point clouds showed much better alignment and ensured consistent accuracy (Fig. 4 to Fig. 5). This is a presentation of some of the results from practical tests.

4.2.4 The fourth experiment

The experiment is designed to study efficiency of the algorithm and iterative convergence.

The efficiency of the algorithm can be measured by time consuming. Since the algorithm do not use methods such as KD tree or other complex “corresponding points” search strategy, using the simple “corresponding points” policy (see 2.2), and therefore the implementation of the algorithm depends on the time consumption of building TIN for the raw point cloud, because the least squares iterative calculation itself is very small; addition this paper, an efficient algorithm for the construction of TIN, the higher efficiency of the algorithm, as shown in Table 6, although a small amount of data, the proposed algorithm and TerraMatch quite efficient, but also slightly lower than TerraMatch, but when a large amount of data, the superiority of the efficiency of the algorithm is particularly evident.

The iterative convergence can be measured by the number of iterations. As shown in Table 6, the number of iterative convergence is gener-
ally not more than 10 times, so the maximum number of iterations is set to 20 earlier, while TerraMatch software general more iterations.

<table>
<thead>
<tr>
<th>strips</th>
<th>no of strip</th>
<th>time consuming/s</th>
<th>no of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>about 1.5 million</td>
<td>9.13</td>
<td>8.7</td>
</tr>
<tr>
<td>A and B</td>
<td>about 10 million</td>
<td>86.12</td>
<td>229.8</td>
</tr>
</tbody>
</table>

5 SUMMARY AND CONCLUSIONS

While most algorithms used in surface matching of spatial data sets minimized the separation between the surfaces along the z axis. The least squares surface matching algorithm presented in this paper minimizes the normal distances from the points of search surface to the triangular plane facets of the reference surface. With the generalized Gauss-Markov model, the unknown transformation parameters are treated as stochastic quantities using proper a priori weights. This extension of the mathematical model gives control over the estimation parameters.

Two real different data sets are used to validate the method. Profiles of specific features before and after the correction are created to show mismatch and match between different strips. In all experiments, after correction the point clouds show much better alignment; “Measure Match” tool of TerraMatch is used to measure how well flightlines match, “average magnitude” indicator is less than 0.05m for ideal data, while poor quality data the error is slightly larger. TerraMatch is used to automated match of strips from different flight lines, so an accuracy comparison between our algorithm and TerraMatch can be made.

But the approach used in this paper only obtains deformation seven parameters between two overlapping strips and matches the two strips together, if the data are inconsistent local deformation, seven parameters between two overlapping strips are not appropriate for the next two strips, which is data-driven model itself defects; At last, accuracy of our algorithm should be improved and we do not consider simultaneous coregistration of multiple 3D pointclouds, these are next to research.

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REFERENCES

一种机载LiDAR条带平差最小二乘匹配算法

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摘 要：基于七参数正形变换的数据驱动模型实现了机载LiDAR条带平差，算法借鉴了Robert(2004)的最小二乘表面匹配思想，通过引入高斯-马尔科夫模型改进了原有算法，得到未知参数的最小无偏方差估计。实验采用两组测试数据，分别考察了引入高斯-马尔科夫模型的必要性，算法效率以及迭代收敛性和算法精度。实验表明：(1)剖面检查吻合且精度一致；(2)TerraMatch量测匹配精度，理想数据高程匹配误差小于0.05 m，数据质量不理想时误差稍大，但仍能成功匹配。

关键词：激光雷达条带平差，最小二乘匹配(LSM)，高斯-马尔科夫模型

1 引 言

激光雷达(Light Detection and Ranging, LiDAR)是一种集激光、全球定位系统(GPS)和惯性导航系统(Inertial Navigation Systems, INS)技术于一身的用于获取地面及目标三维空间信息的主动式雷达探测技术系统。尤其是机载激光雷达测量系统能够高效、准确地测量地形信息，从而为获取高分辨率的地球空间信息提供了一种有效的技术手段，受到了国内外研究机构的极大关注。

机载LiDAR作业时，由于航高和扫描视场角的限制，每条条带的图像只能覆盖地面一定的宽度，要完成一定的作业面积就必须飞行多条航线，而且这些航线还必须保持一定的重叠度(10%—20%)，这使得LiDAR条带平差(strip adjustment)成为一个不可避免的问题。条带平差精度直接影响后期处理的精度，因此条带平差问题在数据处理中的地位也凸显了出来。由于LiDAR结合了定位、定向及激光扫描系统进行观测，任一系统具有量测误差或是系统间的不协调都会产生误差，通常，由于误差会导致LiDAR数据不同条带之间重叠部分出现一定差异，这直接影响机载LiDAR技术在测量生产中的应用。国内外有不少学者对影响机载LiDAR测量精度的误差进行了分析(Huisings和Gomes，1998；Crombaghs等，2000；李树楷和薛永祺，2000；Vosselman和Maas，2001；刘经南等，2002；童俊雄，2005；张小红，2007；王成等，2007)。

条带平差的目的在于通过消除或减少不同条带重叠区域之间的差异，从而生成无缝产品，为最终地理空间产品提供质量保证和质量控制。条带平差(童俊雄，2005)即利用条带间重叠区域共轭点位间的差异进行条带变形整正，平差系统中须先定义条带变形参数，并以最小二乘平差理论求出变形参数值进行点云坐标的调整。


“基于传感器检校的条带平差方法”在理论上较“数据驱动条带平差方法”严密，主要考虑消除最大的系统误差源—激光扫描仪和IMU之间的安置角误差，但平差后仍有残留的系统误差需要应用其他条带平差方法消除，因此Morin和El-Sheimy(2002)指出模型参数的选用必须与条带变形的情况吻合，否则平差效果有限。而“数据驱动条带平差方法”相对简单，其利用空间相似转换模式(七参数正形变换模式)描述机载激光雷达条带坐标系统与地面坐标系统的关系，以达到系统误差改正或消除的效果。另外，也可以运用数据驱动模型去消除检校后数据的残留系统误差，因此研究基于数据驱动的条带平差方法是有意义且有必要的。本文尝试基于数据驱动模型实现机载LiDAR条带平差，并验证方法的可行性。

2 最小二乘表面匹配算法
2.1 匹配原理

给定两个表面，一个为参考面$S_1$，另一个为搜寻面$S_2$，二者具有重叠区域$\Omega$。最小二乘表面匹配的任 务在于找到$S_1$和$S_2$之间的正形变换七参数$(\phi, \omega, \kappa)$为旋转参数，$T_x, T_y, T_z$为平移参数，$S$为比例因子，对$S_2$进行转换得到$S'_2$，使$S_1$和$S'_2$之间沿法向距离平方和最小，建立表面匹配的目标方程：

$$\min \sum S^2 \Delta_i d_i$$

式中，$\Delta_i d_i$是两表面对应点沿法向距离，$\alpha_i$是$\Delta_i d_i$的权，取值0或1，用来处理两个表面没有覆盖相同区域的问题。根据最小二乘原理进行迭代求解，就可以完成参考面与搜寻面间匹配。

对参考面$S_1$构建TIN，任意三角剖面方程不妨记为：$z = f(x, y)$，则$S_2$中一点$(x', y', z')$沿法向向$S_1$中三角面片的距离为：

$$D = \frac{|a \cdot x_0 + b \cdot y_0 + c \cdot z_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

式(2)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$D^* = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z$$

$$\frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(3)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$D^* = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z$$

$$\frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(3)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D = D^* - V$$

式(4)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(5)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D = D^* - V$$

式(6)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(7)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D = D^* - V$$

式(8)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(9)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D = D^* - V$$

式(10)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D_0 + \frac{\partial D}{\partial \omega} \Delta \omega + \frac{\partial D}{\partial \phi} \Delta \phi + \frac{\partial D}{\partial \kappa} \Delta \kappa + \frac{\partial D}{\partial T_x} \Delta T_x + \frac{\partial D}{\partial T_y} \Delta T_y + \frac{\partial D}{\partial T_z} \Delta T_z + \frac{\partial D}{\partial \Delta s} \Delta s - V$$

式(11)为非线性，用泰勒公式展开对方程进行线性化得到法向距离$D$的估值$D^*$. 则

$$V = D = D^* - V$$
尔科夫模型，其中$P_{0}$为先验阵。将未知转换参数都视为基于适当的先验权计算得出的随机量，从而可以对估算参数进行控制。基于这种考虑引人另一个系统参数的观测方程：

$$V_{k} = IX + L_{k} \quad P_{k}$$

式中，$I$为单位矩阵；$L_{k}$为有关的系统矩阵观测向量；$P_{k}$为相应的先验权系数矩阵。当$(P_{k})_{ii} \to \infty$表示第$i$个参数看作是常量，而当$(P_{k})_{ii} = 0$表示第$i$个参数可以任意变化。

联合式(7)和式(8)根据最小二乘原理可以得出高斯–马尔科夫模型的最小方差无偏估计参数:

$$\hat{x} = (A^{T}PA + P_{0})^{-1}(A^{T}PL + P_{0}L_{0})$$

$$\hat{v} = AX + L \quad \hat{V}_{k} = IX - L_{k}$$

$$\sigma^{2}_{s} = \frac{(V^{T}PV + V_{k}^{T}P_{k}V_{k})}{r}$$

式(11)中，$\sigma^{2}_{s}$为单位权中误差。$\hat{r}$代表为最小二乘估值；$r = n - u$为多余观测，$n$为观测方程个数，$u$表示转换参数的个数，此处为7。

2.2 “同名点”建立策略

每次迭代，对重叠区域范围内搜寻面$S_{2}$中任一点$I$，判断$I$位于$S_{1}$中(相同平面坐标)那个三角面片内，过$I'$做对应面片的垂线，垂线与面片的交点即为对应的“同名点”。“同名点”对应关系在迭代过程中不断更新。

2.3 粗差剔除

由于遮挡等原因，在迭代过程中建立的“同名点”中一部分为错误的对应关系，不妨称为“伪对应点”。这些“伪对应点”需予以剔除。本文采用了一种简单的方法：基于统计学原理，在每次求解过程中，基于式(2)对同名点间法向距离做统计(如图1)，由图中可以看出有明显的峰值，该峰值出现的位置应该代表两个表面之间的法向距离。偏离真实距离越远则所含误差越大，超出一定的限度即认为是粗差点，本文选用以3倍中误差作为粗差判断阈值：

$$\lambda = 3 \sigma_{0} \quad \text{即:} \quad D_{i} - D < 3 \sigma$$

其中$\sigma$为标准差的估计量。

值得注意的是：迭代收敛前，随着迭代次数的增加，两表面越来越接近，则法向距离分布图在迭代过程中是变化的，由此确定的均值及标准差也随之变化，因而这样确定的粗差阈值能够适应不同的表面变化，具有良好的自适应性。

2.4 重叠条带偏移量确定

本文采用了一个基于搜索的方法。首先在$S_{2}$中基于Moravec算子提取特征点，然后在以特征点为中心的一定邻域范围内移动$S_{1}$，计算二者的相关系数，相关系数最大且大于0.99即认为搜索到一个最佳的概略匹配位置。

2.5 算法优化

为了避免原始点云数据坐标(例如265732.80，3995270.97，63.87)在不同数量级的转换参数之间引起的误差，本文采用如下优化策略：对匹配点云做质心化处理，质心化由于减小了旋转参数的臂长因而简化了匹配算法。具体做法为：基于参考点云计算其质心，平移参考点云和搜寻点云原点至质心，待匹配完成后再转换到原坐标系统下即可。

3 匹配算法流程

3.1 预处理

算法基于地面点完成匹配，所以需首先对原始数据进行滤波分离出地面点。

3.2 近似转换参数求解

基于2.4节所述搜索原理确定重叠条带偏移量作为平移参数的初始值；机载LiDAR数据旋转参数不会太大，初始值设为0；而比例因子基本不变，初始值设为1。
3.3 基于最小二乘匹配算法的条带平差

基于最小二乘匹配算法的条带平差如图2:

![算法流程图](image)

图2 算法流程图

其中，迭代终止条件可以是：(1)最大迭代次数(本文设为20)。(2)相邻两次迭代求得变换参数的差值足够小。7个变换参数的限差值由各个观测值的精度决定。在实际匹配应用中，各个参数的限差选择则依据其对应的观测精度、经验值确定，在此不再做进一步讨论。(3)法向距离$\bar{D}$均值满足$\bar{D} < \bar{D}_0$, $D \in L$，$i$为迭代次数，否则停止迭代。(4)单位权中误差$\sigma_0(i+1) < \sigma_0(i)$，否则停止迭代。

4 实 验

实验平台的配置如下：CPU 为 Intel Xeon E5420 2.50 GHz，内存4 GB，Windows XP系统。

开发平台：Microsoft Visual Studio 2008 C++。

文中匹配精度利用TerraMatch软件的Measure Match工具定量测量。基于平均强度(Average magnitude)，表示两匹配表面中对应点高差的平均值，如果重叠条带之间的高程系统误差已消除，则该值表示剩余随机误差，指标表征精度的高低。

4.1 实验数据

第一组实验数据为TerraSolid软件自带示例数据。第二组实验数据为ALS50-Ⅱ检校数据，但从检校飞行的结果来看，ALS50-Ⅱ系统IMU/DGPS组合导航解算的的姿态、位置参数不够准确。两条航线间的位置偏移最大达到了23 m(采用此数据旨在考察算法在数据质量不理想情况下精度)。如图3所示。数据获取基本情况如表1所示。

$\begin{align*}
\text{数据} & \quad \text{航高/m} & \quad \text{条带数} & \quad \text{点数} & \quad \text{仪器} & \quad \text{编号} \\
1 & 100 & 4 & 312089 & \text{TopEye} \\
2 & 1350/2300 & 4 & 5669834 & \text{ALS50-Ⅱ} \\
\end{align*}$

4.2 实验分析

4.2.1 实验1

考察不采用高斯-马尔科夫模型，以两条条带中不同条带为参考面的匹配精度情况，如表2所示。其中“1和2”在前表示1为参考面，2为搜寻面，其他

$\begin{align*}
\text{匹配条带} & \quad \text{平均强度/m} \\
1\text{和2} & \quad 0.05021 \\
2\text{和1} & \quad 0.03414 \\
3\text{和4} & \quad 0.02298 \\
4\text{和3} & \quad 0.03108 \\
\end{align*}$
类推。

由表3可知：条带两两匹配时，以不同条带为参考面其匹配精度不同，有高低之分，以条带1和条带4匹配实验为例，条带1为参考表面时高程匹配精度为0.04521,而条带4为参考表面时高程匹配精度为0.03646。分析认为，其精度不同可能与重叠区域的点云密度有关，因为对参考面S构造TIN的正确性随着对真实表面采样密度的增加而增加，而内插误差的大小也与S的密度有关，因此参考面密度越大条带平差精度应越高。可以参阅表3重叠区域点云密度情况，对比表4与表3不难发现精度高低与密度大小吻合。因此在平差程序中只需以点云密度大的表面为参考面即可。

4.2.2 实验2

为考察引入高斯-马尔科夫模型的必要性，设计实验如下：如表4所示，估算参数依次TX, TY, TZ, S, φ, κ, P为七参数的先验权系数矩阵。P的设置基于这种考虑：机载LiDAR数据旋转角不会太大，初始值和对应权值总赋予0,认为此参数可以任意变化；只需考察其余四者权值的变化对匹配精度的影响，此在认为初始平移参数和比例因子等权，分别考虑0, 10, 100, 1000, 10000五种情况。

由表4可知：(1)对比估算参数变化情况可以看出权值的大小对匹配精度的影响：当权值设置为0时，对应参数变化幅度最大；随着权值的增大(0—10000)参数变化的幅度也越来越小，当权值为10000时对应的参数基本不变，因而可以改变权值的大小达到对估算参数控制的目的。(2)对比表中平均强度指标可以看出：只有设置合适的权值才能达到既对估算参数进行控制同时又提高匹配精度的目的，同时这也可证明引入高斯-马尔科夫模型的必要性。

然而P的确定比较困难，因为不同数据需设置可能不同并且合适的P才能达到提高精度的目的。因此本文设计了自适应确定P方案：根据方差的大小确定合适的权值。即每次迭代，分别考虑P取表5五种情况(可以改变)下，依式(9)、式(10)、式(11)计算单位权中误差大小，取单位权中误差最小值为当次迭代的P值。

4.2.3 实验3

考察算法精度。

<table>
<thead>
<tr>
<th>点数</th>
<th>1-11625</th>
<th>2-19606</th>
<th>3-15802</th>
<th>4-18127</th>
<th>1-11146</th>
<th>2-19603</th>
<th>3-54367</th>
<th>2-54474</th>
</tr>
</thead>
<tbody>
<tr>
<td>表3 重叠区域点云密度情况</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>条带</td>
<td>1和2</td>
<td>3和4</td>
<td>1和4</td>
<td>2和3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>点数</td>
<td>1-11625</td>
<td>2-31802</td>
<td>4-18127</td>
<td>1-11146</td>
<td>2-19603</td>
<td>3-54367</td>
<td>2-54474</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P_1</th>
<th>估算参数</th>
<th>平均强度/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0,0,0</td>
<td>1.0493,-1.3938,-0.7628,0.99663,0.01053,-0.00283,-0.00120</td>
<td>0.02298</td>
</tr>
<tr>
<td>0.1,0,0</td>
<td>1.0239,-1.3822,-0.7479,0.99696,0.010422,-0.002850,-0.00134</td>
<td>0.02283</td>
</tr>
<tr>
<td>1.0,0,0,0</td>
<td>0.9823,-1.3802,-0.6858,0.998058,0.00955,-0.00294,-0.00131</td>
<td>0.03186</td>
</tr>
<tr>
<td>10.0,0,0,0</td>
<td>1.0037,-1.5077,-0.7884,0.999294,0.010481,-0.00280,-0.00131</td>
<td>0.02561</td>
</tr>
<tr>
<td>100,0,0,0,0</td>
<td>1.00006,-1.49902,-0.0589846,0.999294,0.00225175,-0.00366645,-0.000637851</td>
<td>0.0940</td>
</tr>
</tbody>
</table>

| 表4 权值对参数影响 |

| 表5 第一组数据匹配精度对比 |

<table>
<thead>
<tr>
<th>条带</th>
<th>原始数据/m</th>
<th>本文算法/m</th>
<th>TerraMatch/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>2和1</td>
<td>0.09471</td>
<td>0.09779</td>
<td>0.09831</td>
</tr>
<tr>
<td>4和3</td>
<td>0.03093</td>
<td>0.02283</td>
<td>0.03601</td>
</tr>
<tr>
<td>4和1</td>
<td>0.01832</td>
<td>0.02579</td>
<td>0.01955</td>
</tr>
</tbody>
</table>
表6 第二组数据匹配精度对比

<table>
<thead>
<tr>
<th>条带</th>
<th>A和B</th>
<th>C和D</th>
<th>A和C</th>
<th>B和D</th>
</tr>
</thead>
<tbody>
<tr>
<td>原始数据/m</td>
<td>0.0977</td>
<td>0.0979</td>
<td>0.1049</td>
<td>0.1363</td>
</tr>
<tr>
<td>本文算法/m</td>
<td>0.04984</td>
<td>0.06088</td>
<td>0.0548</td>
<td>0.05255</td>
</tr>
<tr>
<td>TerraMatch/m</td>
<td>0.0342</td>
<td>0.0481</td>
<td>0.03652</td>
<td>0.05021</td>
</tr>
</tbody>
</table>

综合表5、表6可知：(1)定量测量匹配精度，理想数据高程匹配误差小于0.05 m，数据质量不理想时稍大；(2)从平均强度这一项指标来看，本文算法和TerraMatch软件相比总体上精度偏低，如表6中“A和B”匹配实验中本文算法平均精度为0.04984 m，而TerraMatch软件的则为0.0342 m。但本文算法在以上所有实验中均成功完成所有匹配，表现出较好的稳定性，从而也验证了算法的可行性。可通过剖面图查看其数据吻合情况，如图4为第一组数据匹配后精度最差的“4和1”斜坡剖面图；图5为第二组数据匹配后精度最差的“C和D”屋顶剖面图；(3)由表6可知第二组实验数据匹配精度较低，分析原因可能是ALS50-Ⅱ系统IMU/DGPS组合导航解算的姿态位置参数不够准确造成的数据局部变形不一致，而算法仅采用七个参数不足以描述其变形情况，条带平差的效果有限造成的，此时平差模型需要更新，采用九参甚至十二参。因此，当实际的条带系统误差具有变形量则必须加入额外的参数表示变形状况，如Kilian(1996)采用平移、旋转以及1次多项式的平移与旋转线性变化参数共12个条带变形参数改正系统误差造成的条带变形，Vosselman and Mass(2001)则采用了9个条带变形参数；反之，如果过量使用参数，则可能因此在平差结果中产生其他误差(Filin, 2003)。

4.2.4 实验4

考察算法效率与迭代收敛性。

算法的执行效率可以通过算法执行消耗的时间来衡量。由于本文算法未采用诸如八叉树法、空间单元格法、k-d树法等“同名点”搜索策略，采用了简单的“同名点”建立策略，因而算法的执行消耗的时间主要取决于对原始点云构建TIN的时间，因为最小二乘迭代本身计算量较大，本文算法和TerraMatch软件的效率相当。当大据量时，虽然本数据集的效率低于TerraMatch，但是当大数据量时，本文算法的效率优势性特别明显。

迭代收敛性可由正确完成匹配所需要的迭代次数来衡量。如表7所示，实验表明：本文算法收敛次数一般不会超过10次，因此前文中最大迭代次数设置为20，而TerraMatch软件迭代次数一般较多。
表 7 算法效率和迭代收敛性

<table>
<thead>
<tr>
<th>条带</th>
<th>重叠点数</th>
<th>消耗时间/秒</th>
<th>迭代次数</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 和 2</td>
<td>约 1.5 万</td>
<td>9.13</td>
<td>8.7</td>
</tr>
<tr>
<td>A 和 B</td>
<td>约 10 万</td>
<td>86.12</td>
<td>229.8</td>
</tr>
</tbody>
</table>

5 结论与展望

本文基于七参数正形变换的数据驱动方法实现了机载 LiDAR 条带平差，算法借鉴并改进了 Robert(2004)的最小二乘匹配算法，通过引入高斯—马尔科夫模型，得到了未知参数的最小无偏方差估计。最后采用两组实测数据评估了算法的正确性及精度，实验表明：(1) 幅面检查吻合且精度一致；(2) TerraMatch 量测匹配精度，理想数据高程匹配误差小于 0.05 m，数据质量不理想时稍大；(3) 仅从平均精度指标来看，本文算法精度和 TerraMatch 软件相比稍低，但从数据质量幅面检查及定量测量结果来看，算法均能成功完成匹配且精度一致，定量测量结果表明其精度完全满足实际生产需求，从而证明了这种算法的可行性。

本文所采用数据驱动方法只是求出了两条条带之间的形变七参数，将两个条带匹配在一起，对于数据局部变形不一致，变形参数不足以描述变形情况，条带平差的效果可能有限，此时平差模型需要更新，而本文并未考虑这种情况，需完善；其次，两条条带之间的形变参数放到下两条条带之间时就不再适用，这也是数据驱动模型本身存在的缺陷；另外，如果 LiDAR 数据校检完毕效果不理想，可以应用本文算法消除残余系统误差，使条带间完全匹配。

志谢 此次实验数据由中国测绘科学研究院航空遥测所提供，在此表示衷心的感谢！

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